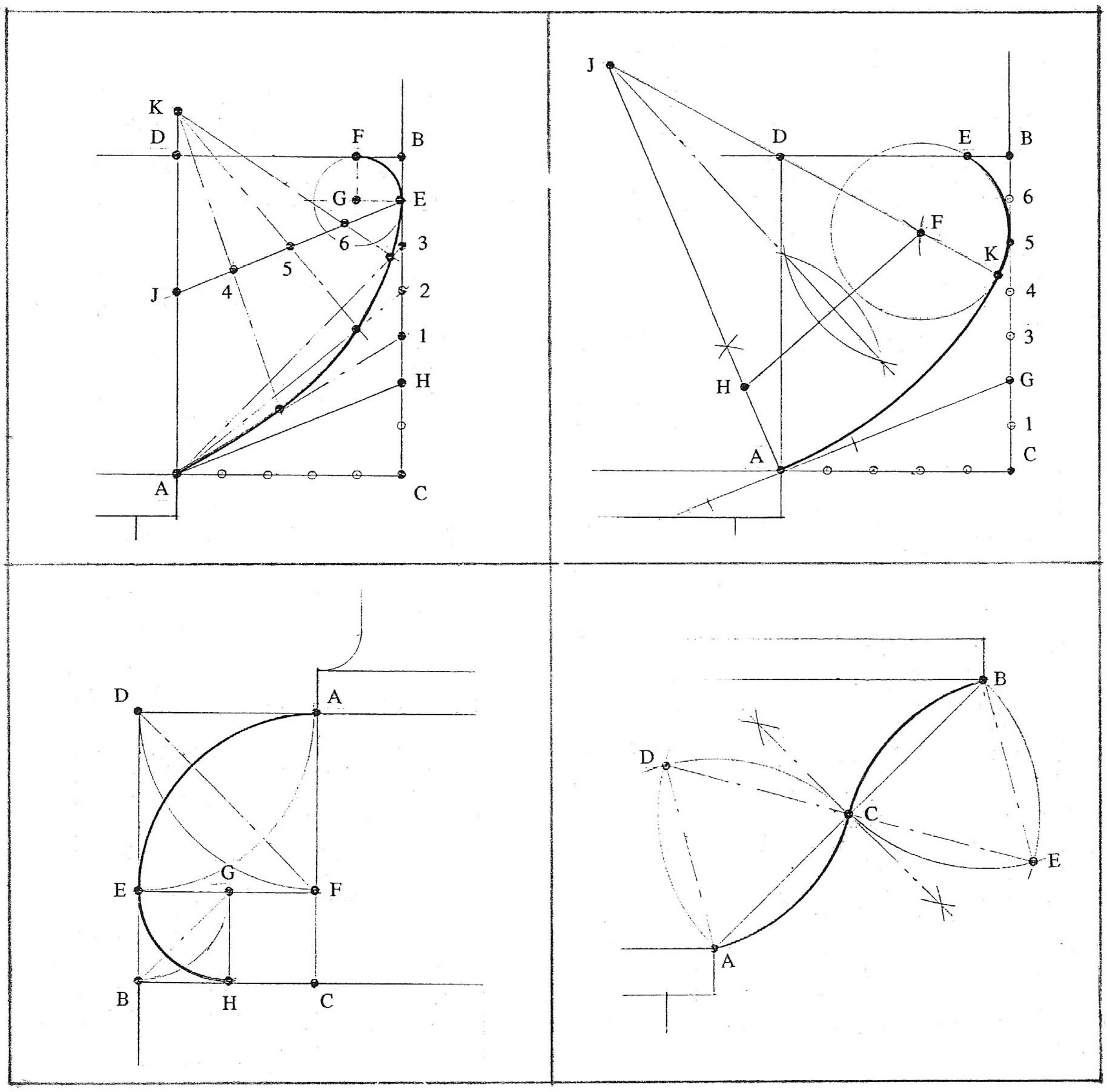
Constructive Geometry



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Introduction

Geometry is said by Herodotus to have begun in Egypt where the annual flooding of the Nile required the re-surveying of the fields. Thus the term geo-metry, which in Greek means earth-measure. It is largely on this practical level that these present notes are directed.

This material is offered in the hope that it will be an assist to designers and artisans in their actual work. The student should attempt to recreate these exercises manually, that is, to actually draw them. This helps to establish a mind-body connection and develop the manual skills necessary to create objects in the classical aesthetic.

But geometry is more than the measuring of the physical. Because it utilizes the method of logic it stimulates intellectual clarity. This, at least, allows the designer to make rationally based choices with awareness of alternatives and possible outcomes.

Taking us beyond the logical, Plato asserts that the study of geometry, when *"pursued for the sake of knowledge … compels us to contemplate reality rather than the realm of change."* Since *" … the objects of* [geometrical] *knowledge are eternal and not liable to change and decay"*, such study *"will tend to draw the mind to the truth and direct the philosophers' reason upwards* …" [Republic 527].

Material here focuses on examples that occur in design such as the construction of the polygons and in detailing, such as the construction of moulding profiles. By including examples that appear in the work of Serlio, Palladio and others this presentation may also be seen as preparation for approaching the cannonical literature of classical architecture.

The focus here is on what might be called 'constructive' rather than theoretical geometry. For this reason, no proofs are offered. Preliminary definations and constructions, such as bisecting lines, are kept to a minimum. Since there were six presentations each with its own theme the examples have been numbered approximately in the order they were presented for the convience of the student. References to prior constructions are indicated in parenthises [ ].

We used as references Heath's translation of Euclid augmented by an unpublished manuscript 'Handbook of Practical Geometry' by Bob Greenberg of Ryerson College, Toroto, 1982 furnished by Richard Cameron; and 'Engineering Drawing' by Thomas E. French, New York, 1947 which my father had used when he studied the subject in the 1950's.

S. B. 2013

Definitions

The following definitions are limited to terms used in the constructions and commentary.

point - A point is that which has no parts; or it may be said that a point has locaation without dimension.

line - A line has length but no other dimensions.

plane - A plane surface is that which has length and width only.

straight line - A straight line is the shortest distance between two points; or it may be saidthat any part of it fitson any other part regardless of orientation.

parallel lines - Parallel lines are the same distance from one another at any given point. Parallel lines may be extended indefinitely on a plaane surface without meeting.

angle - A plane angle is the inclination to one another of two lines in a plane surface which meet one another. A right angle is formed when the angles of intersection of the lines are equal. An angle which is less than a right angle is called acute; an angle which is more than a right angle is called obtuse.

perpendicular - A perpendicular is a line which meets another line at a fight angle.

circle - A circle is a plane figure contained by one line, called the **circumference**, on which all points are equidistant from a single point, not on the line, called the center.

diameter - A straight line throught the center of a circle terminated on both ends by the circumference.

radius - A straight line from the center of a circle to the circumference. The radius equals one-half the diameter.

arc - An arc is a portion of the circumference of a circle. A **chord** is a straight line joining two points on an arc.

polygon - A polygon is a plane figure bounded by three or more straight line sides. A **regular polygon** is a figure made of some number of straight lines of equal length whose interior angles are equal. A regular polygon can be inscribed in a circle. All the vertices of a regular polygon fall on the same circle.

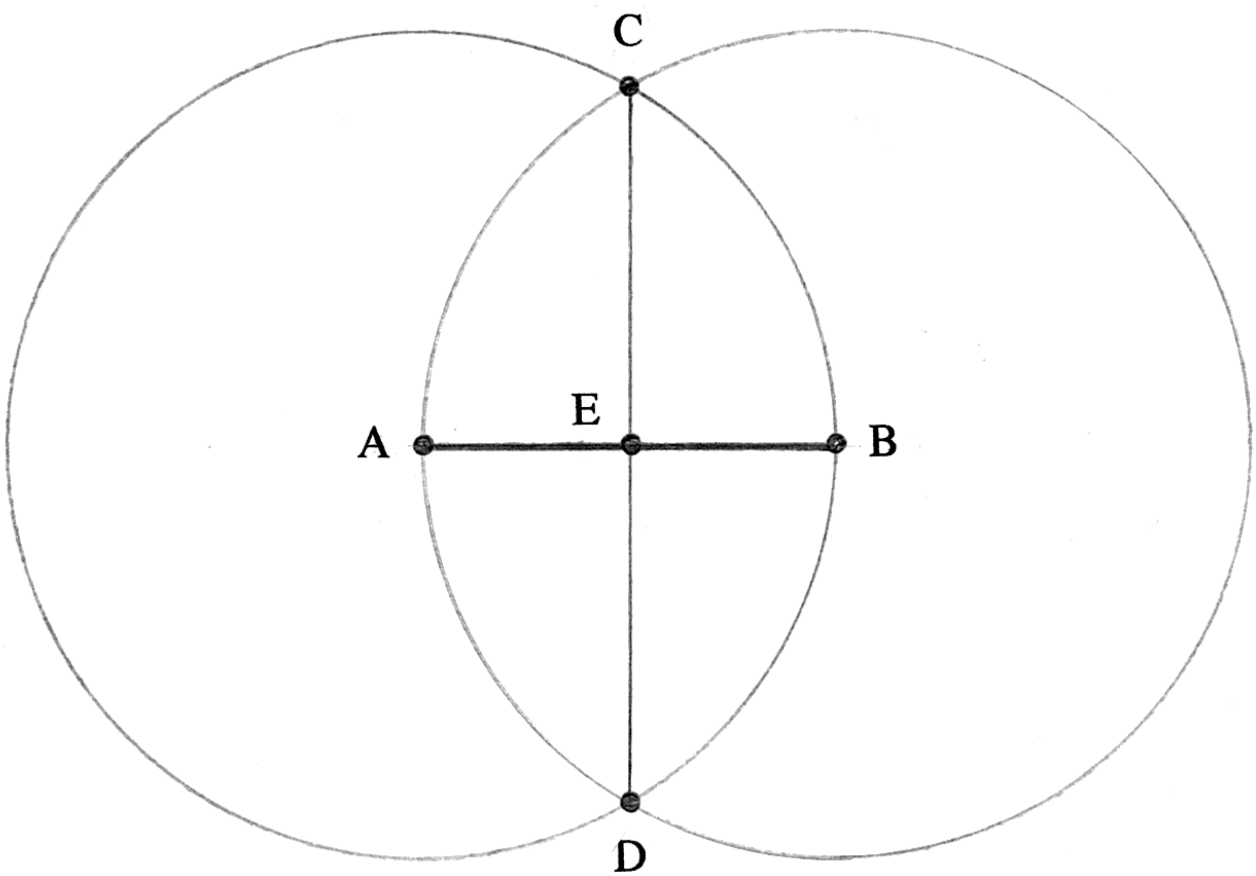
triangle - A triangle is a plane figure enclosed by three straight lines. An **equilateral triangle** is a triangle all three of whose sides and interior angles are equal.

square - A square is a regular polygon with four equal sides whose interior angles are all right angles.

rectangle - A rectangle is a figure contained by four straight lines whose adjacent sides are unequal but whaose interior angles are all right angles.

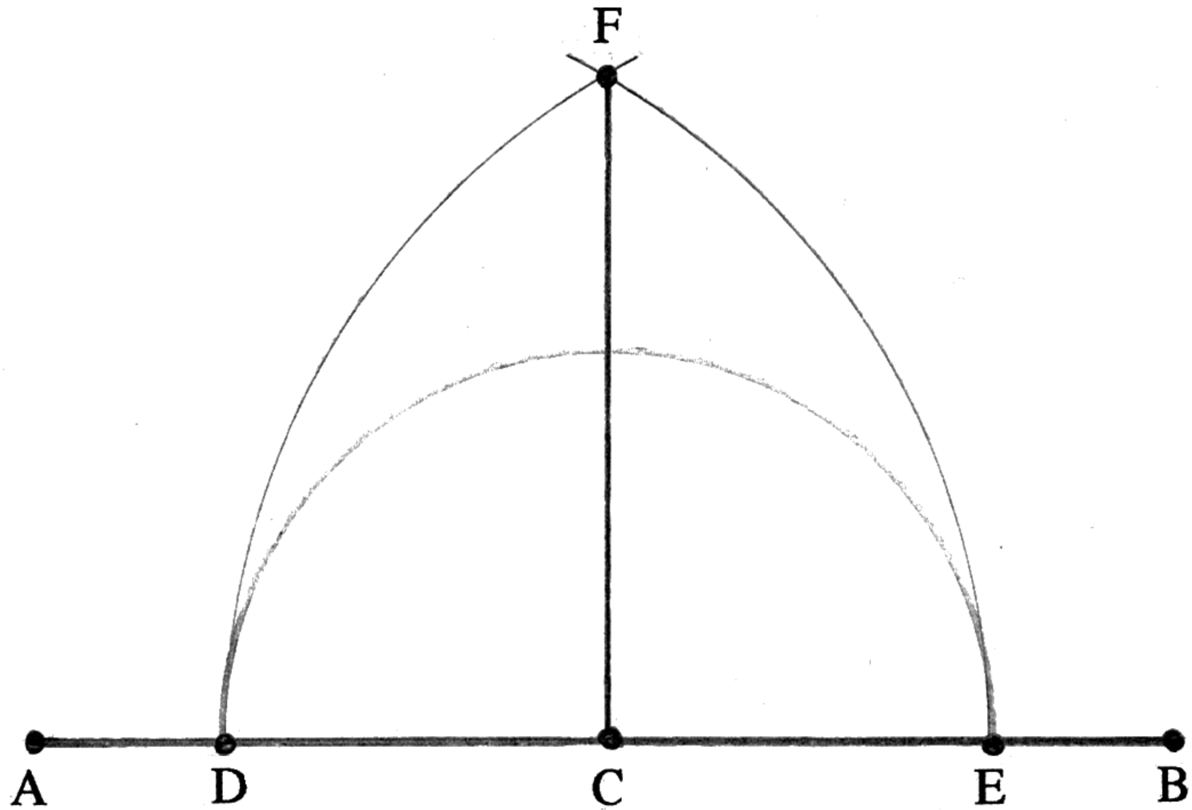
bisect - To divide a line, angle or area in half.

1.1 Bisect a line.



1. Given line AB.
2. Draw circles with centers at A and B, and radius AB to find points C and D.
3. Draw line CD to find point E.
4. E is the center of AB.

1.2 Construct a perpendicular from a point on a line.



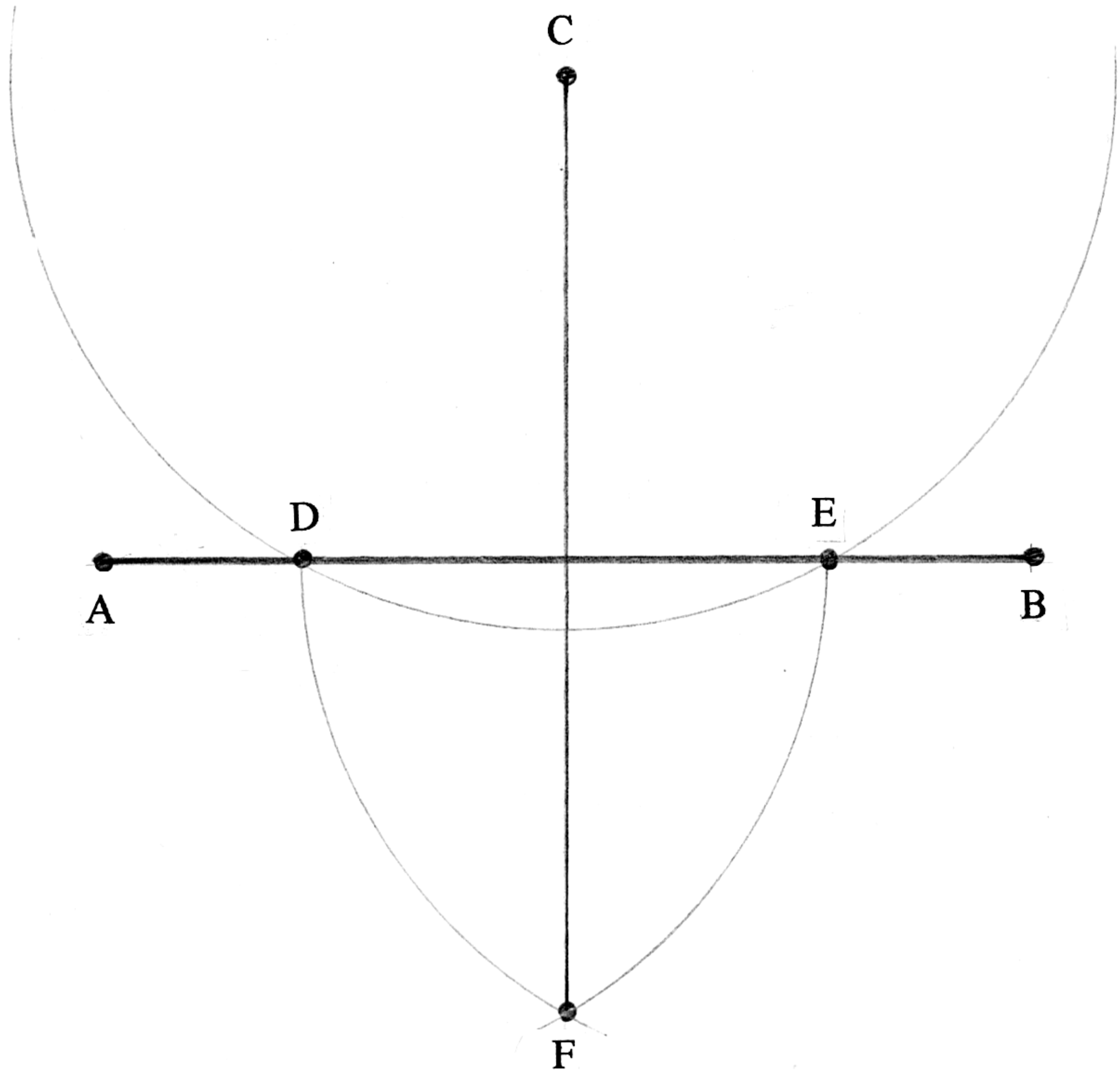
1. Given line AB with any point C.

2. Draw a circle of any convenient radius with center at C to find points D and E.

3. Draw arcs with centers at D and E, radius DE to find point F.

4. Draw line CF. CF is perpendicular to AB

1.3 Construct a perpendicular to a given line from a point off the line.



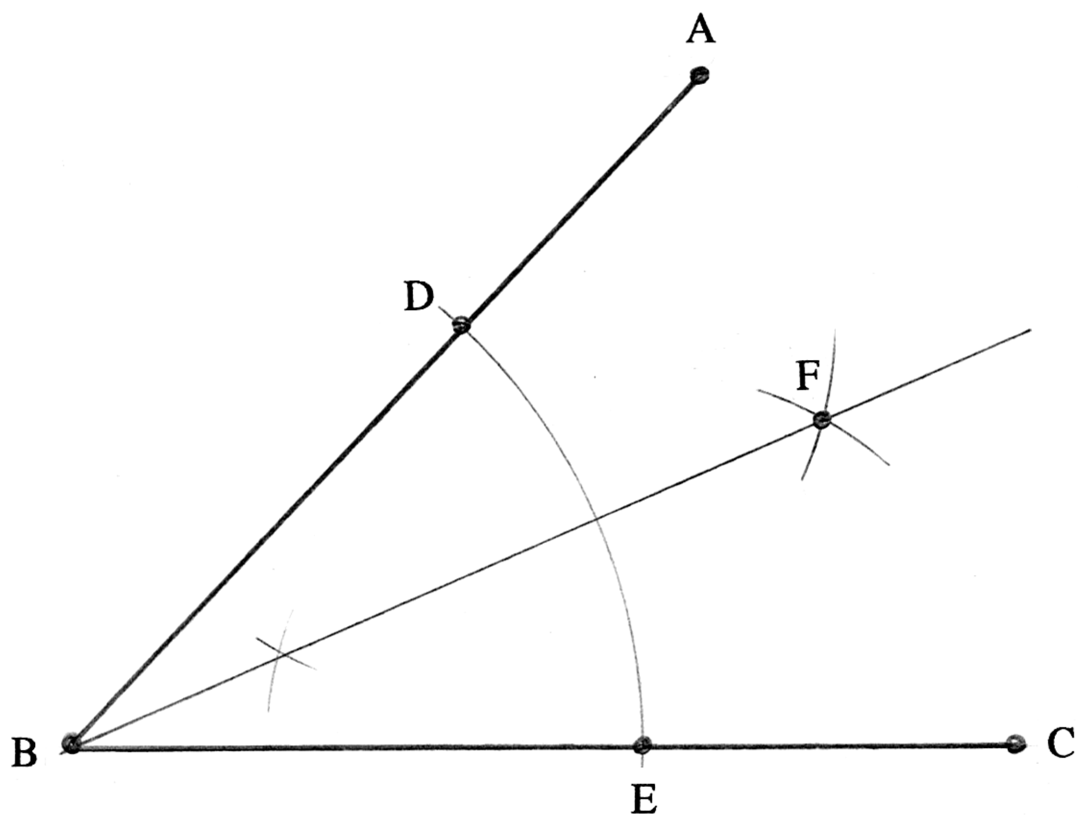
1. Given line AB and point C.

2. Draw a circle with center at C and any convenient radius large enough to intersect AB, to find points D and E

3. Draw arcs with centers at Dand E and radius DE to find point F.

4. Draw line FC. FC is perpendicular to AB.

1.4 Bisect an angle.



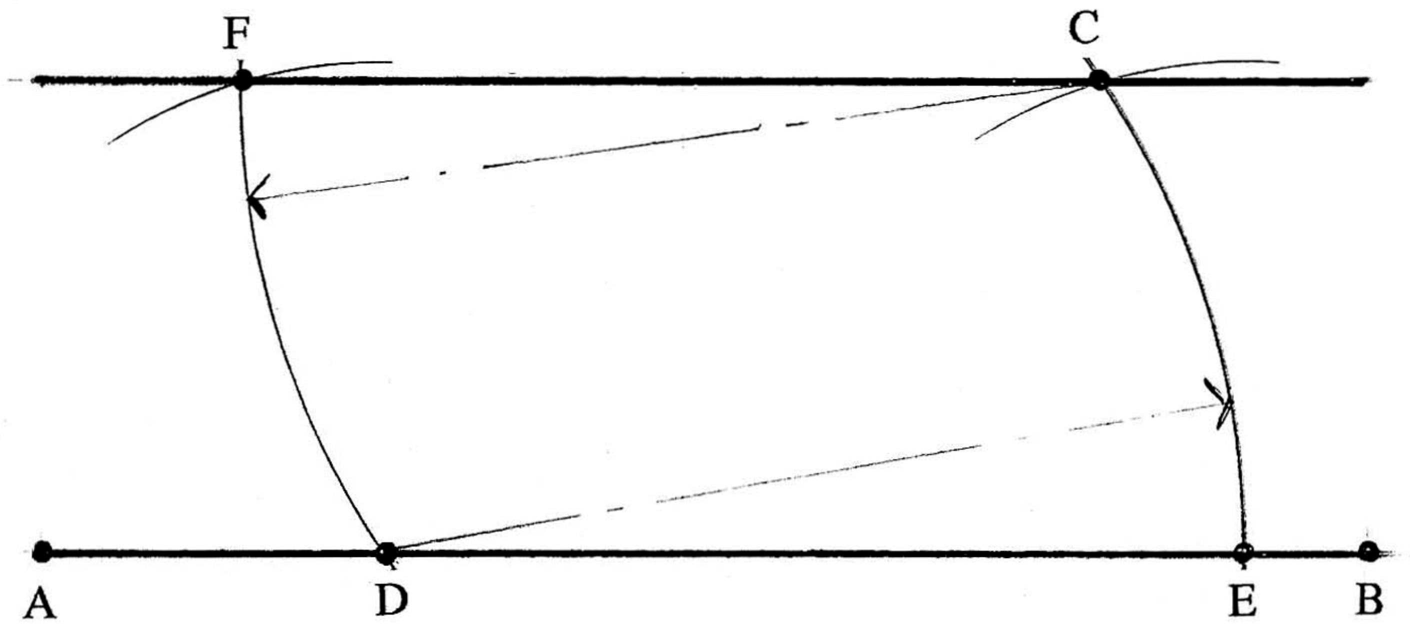
1. Given angle ABC.

2. Draw an arc with center at B and any convenient radius to find points D and E.

3, Draw arcs with centers at D and E, and any convenient radius to find point F.

4. Draw line BF. Angle ABF = FBC = one half of ABC.

1.5 Construct a line parallel to a given line.



1. Given line AB and point C.

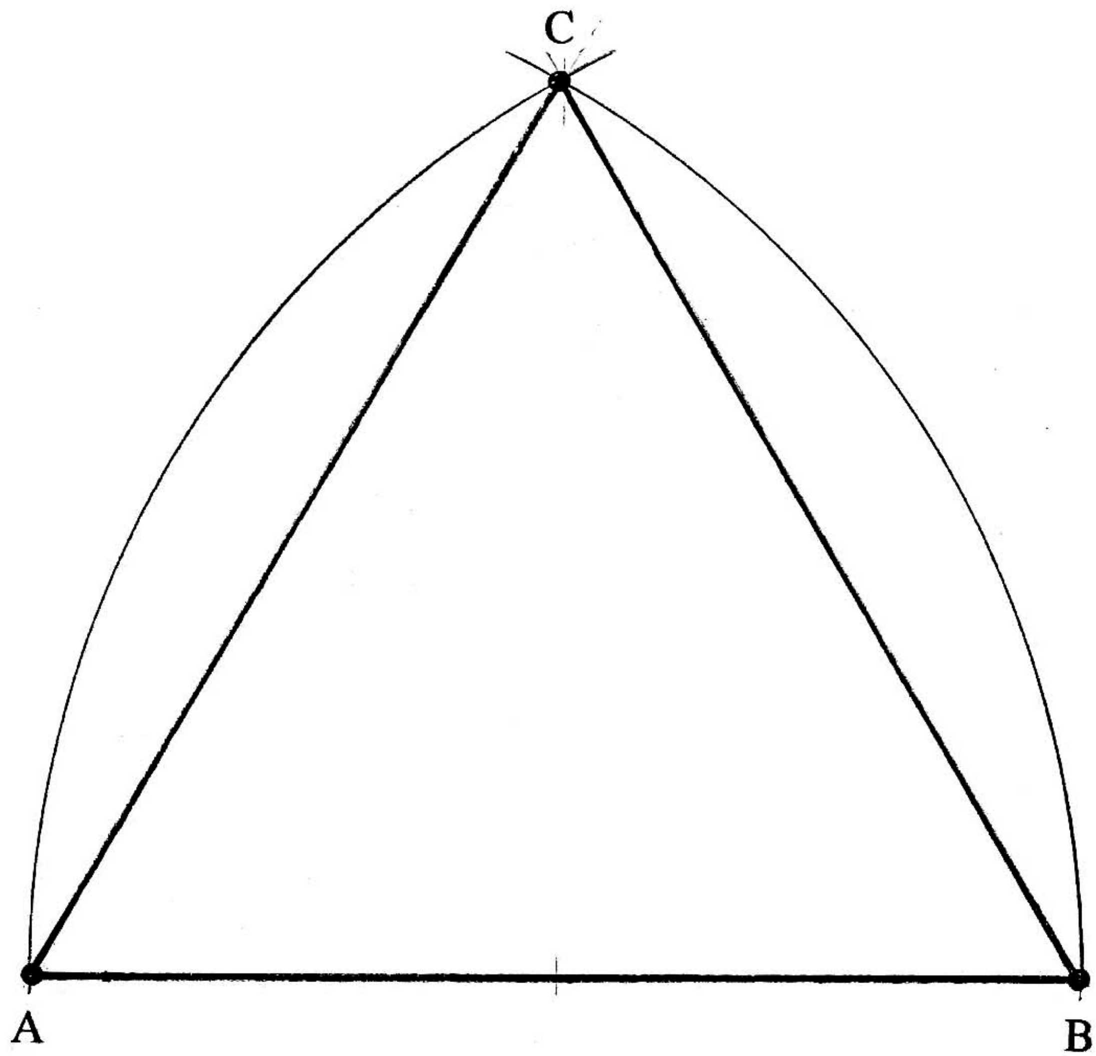
2. Draw an arc with center at C and any convenient radius to find pointD.

3. Draw an arc with center at D and radius CD to find point E.

4. Draw an arc with center at D and radius CE to find point F.

5. Draw line FC. FC is parallel to AB.

1.6 Construct an equilateral triangle given a side.

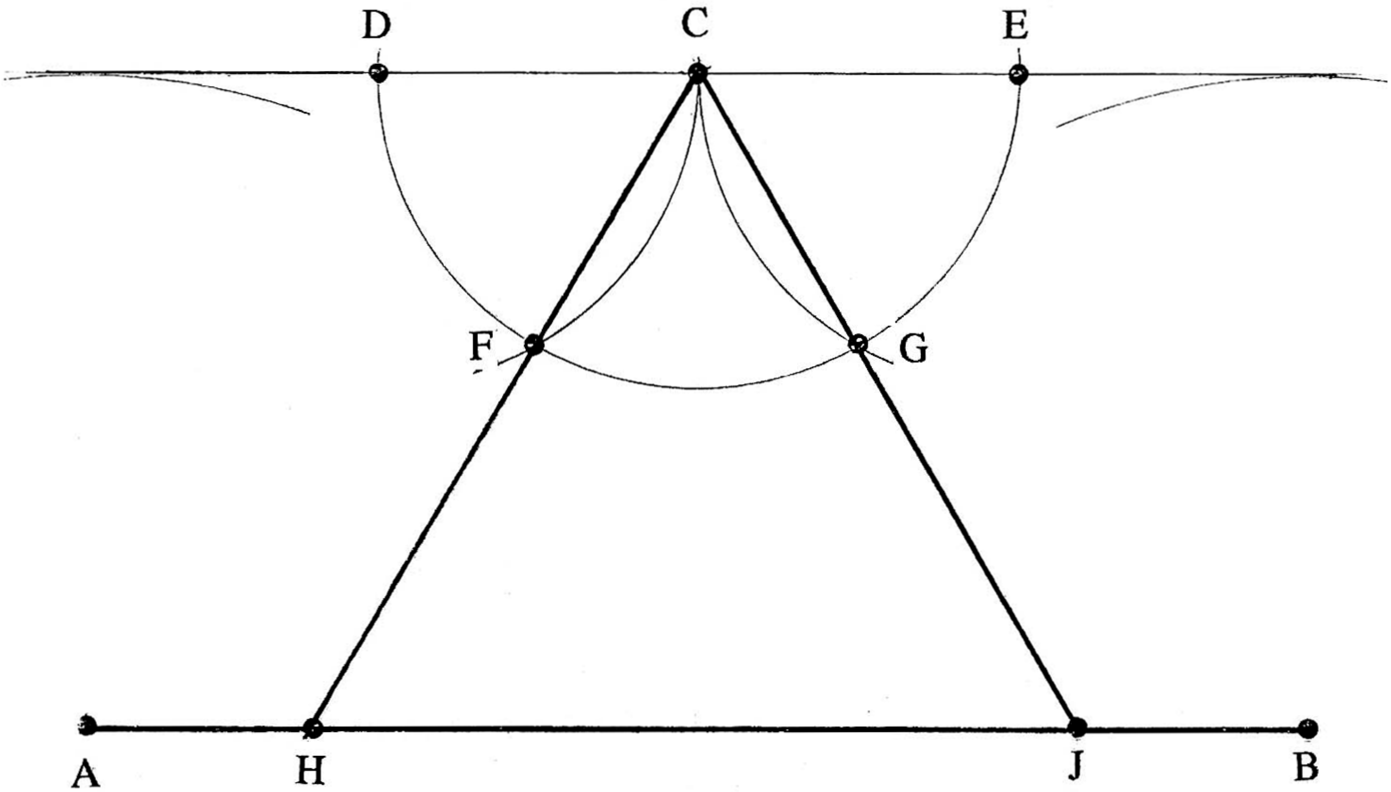


1. Given line AB

2. Draw arcs with centers at A and B, and radius AB to find poit C.

3. ABC is an equilateral triangle.

1.7 Construct an equilateral triangle given the altitude.



1. Given line AB and point C any distance from AB

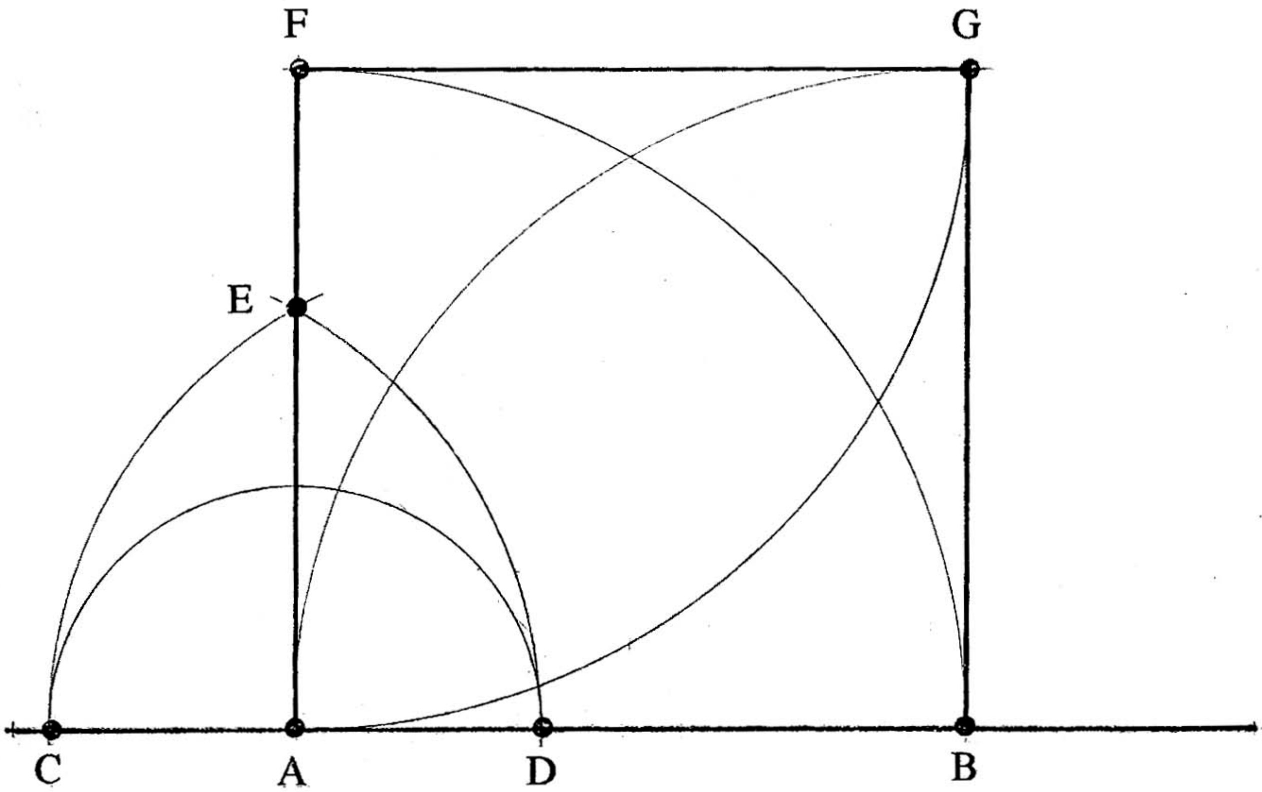
2. Construct a line parallel to AB through C. [1.5]

3. Draw a circle with center at C and any convenient radius to find points D and E.

4. Draw arcs with centers at D and E and radius DC to find points F and G.

5. Draw lines from C through F and G to find points H and J. CHJ is an equilateral triangle.

2.1 Construct a square given a side.



1. Given interval AB on a straight line.

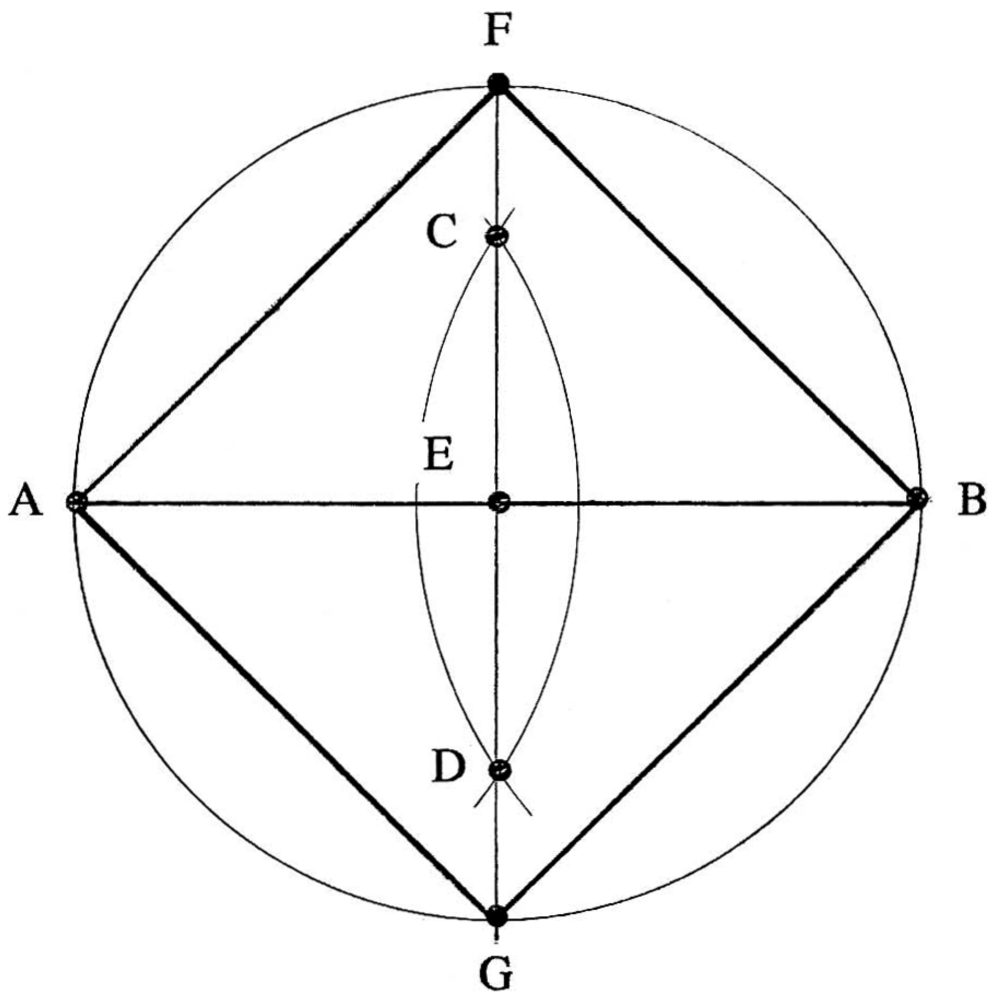
2. Draw circle with center at A and any convenient radius to find points C and D

3. Draw arcs with centers at C and D, radius CD to find point E. Draw line AE.

4. Draw arc with center at A and radius AB. Extend AE to find point F.

5. Draw arcs with centers at B and F, radius AB, to find G. ABFG is a square.

2.2 Construct a square given its diagonal



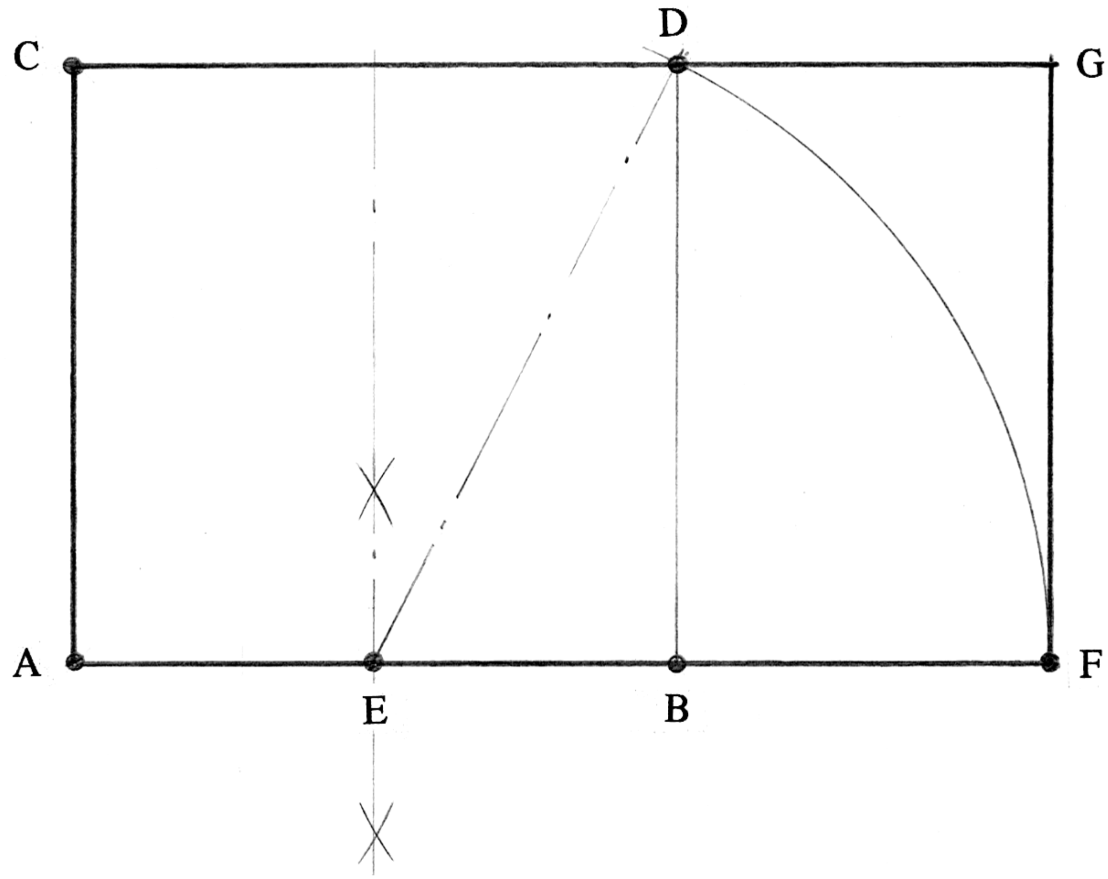
1. Given line AB

2. Draw arcs with centers at A and B at
any convenient radius to find points C and D. Draw line
CD to find point E.

3. Draw a circle with center at E and radius AE.

4. Extend CD to find points F and G. ABFG is a square.

2.3 Construct a rectangle whose sides are in Golden Section relation.



1. Given line AB.

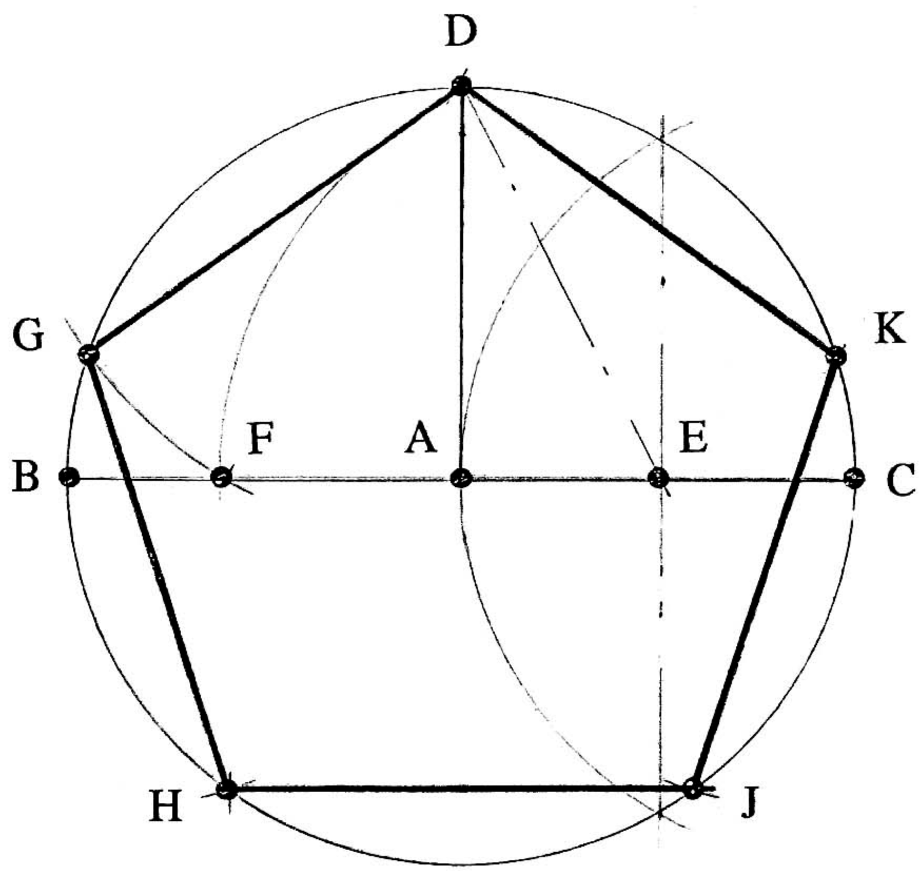
2. Construct square ABCD. [2.1]

3. Bisect AB to find E. [1.1] Draw ED.

4. Draw arc with center at E and radius ED
   to find point F on the extension of AB.

5. Construct a perpendicular on point F [1.2] and
   extend CD to find point G. AC:AF = 1:1.618…

2.4 Construct a pentagon within a circle.



1. Given a circle with center at A and diameter AC.

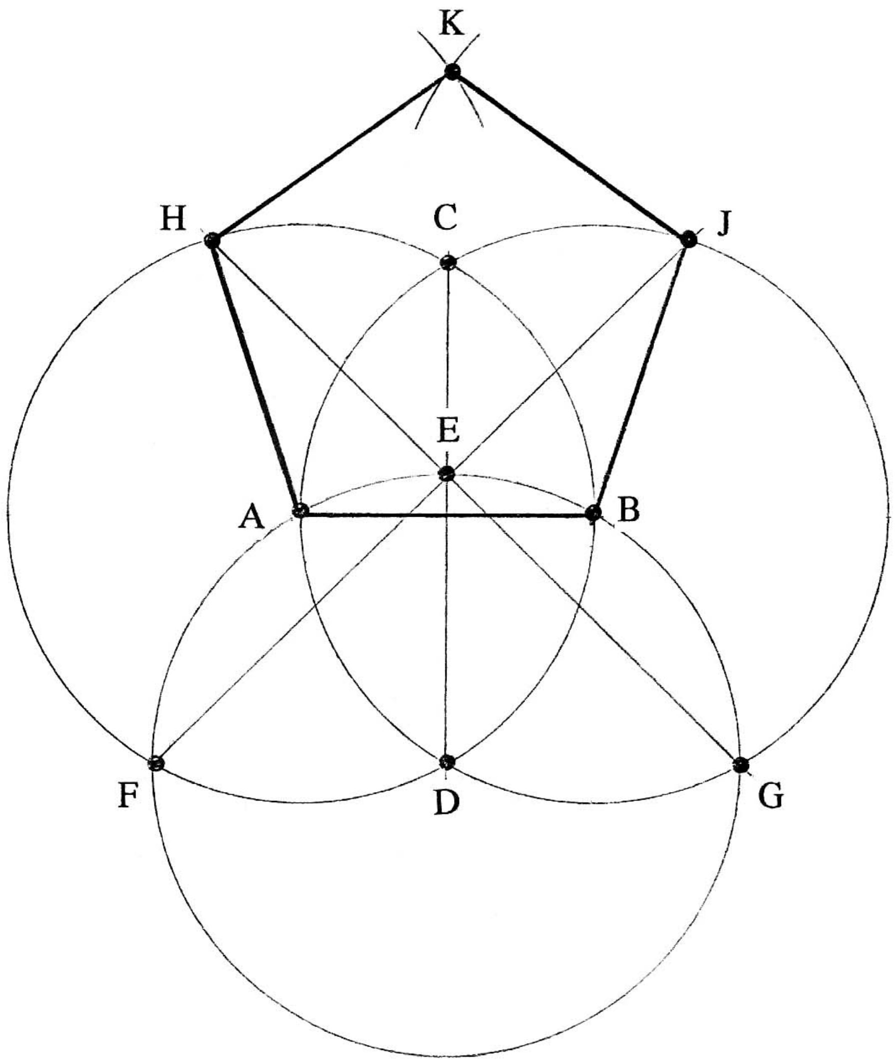
2. Erect a perpendicular from A to find D. [1.2]

3. Bisect AC to find E. [1.1]

4. Draw arc with center at E and radius ED to find point F. Draw arc with center at D and radius DF to find point G.

5. Mark off radius DF = DG on the circumference to find points H, J and K. DGHJK is a pentagon.

2.5 Construct a pentagon given a side.



1. Given line AB.

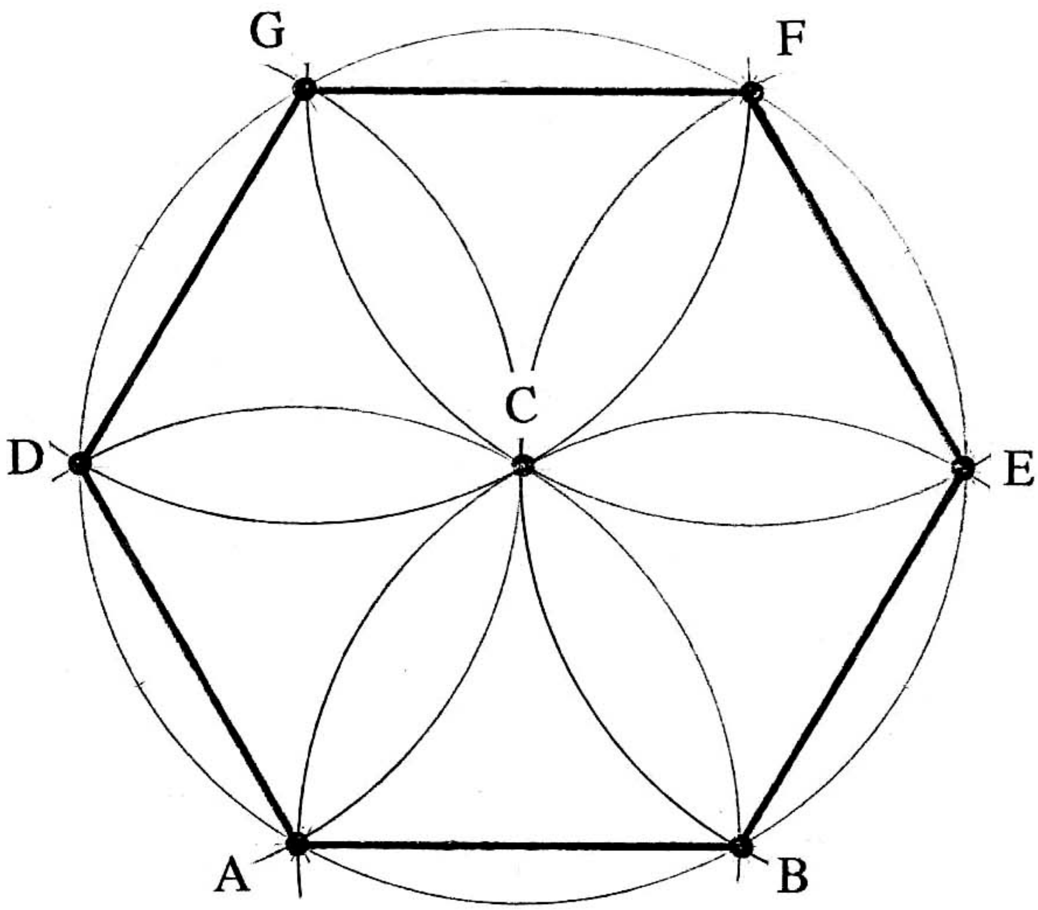
2. Draw circles with centers at A and B, and radius AB to find points C and D. Draw line CD.

3. Draw a circle with center at D and radius AB to find points E, F and G

4. Draw line EG and extend to H. Draw EF and extend to J.

5. Draw arcs with centers at H and J, radius AB to find K. ABJKH is a pentagon.

3.1 Construct a hexagon given a side.



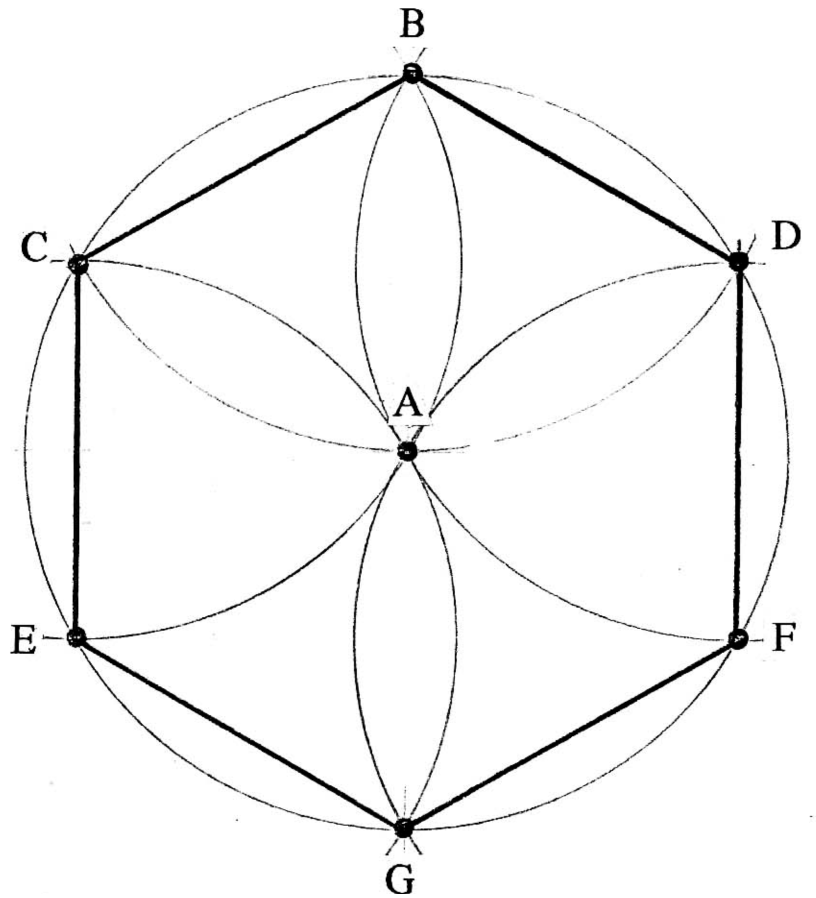
1. Given line AB.

2. Draw arcs with centers at A and B, and radius AB to find point C.

3. Draw circle with center at C and radius AB = AC. Extend arcs AC and BC to find points D and E.

4. Draw arcs wiith centers at D and E. radius AB to find points F and G. ABDEFG is a hexagon.

3.2 Construct a hexagon within a circle.



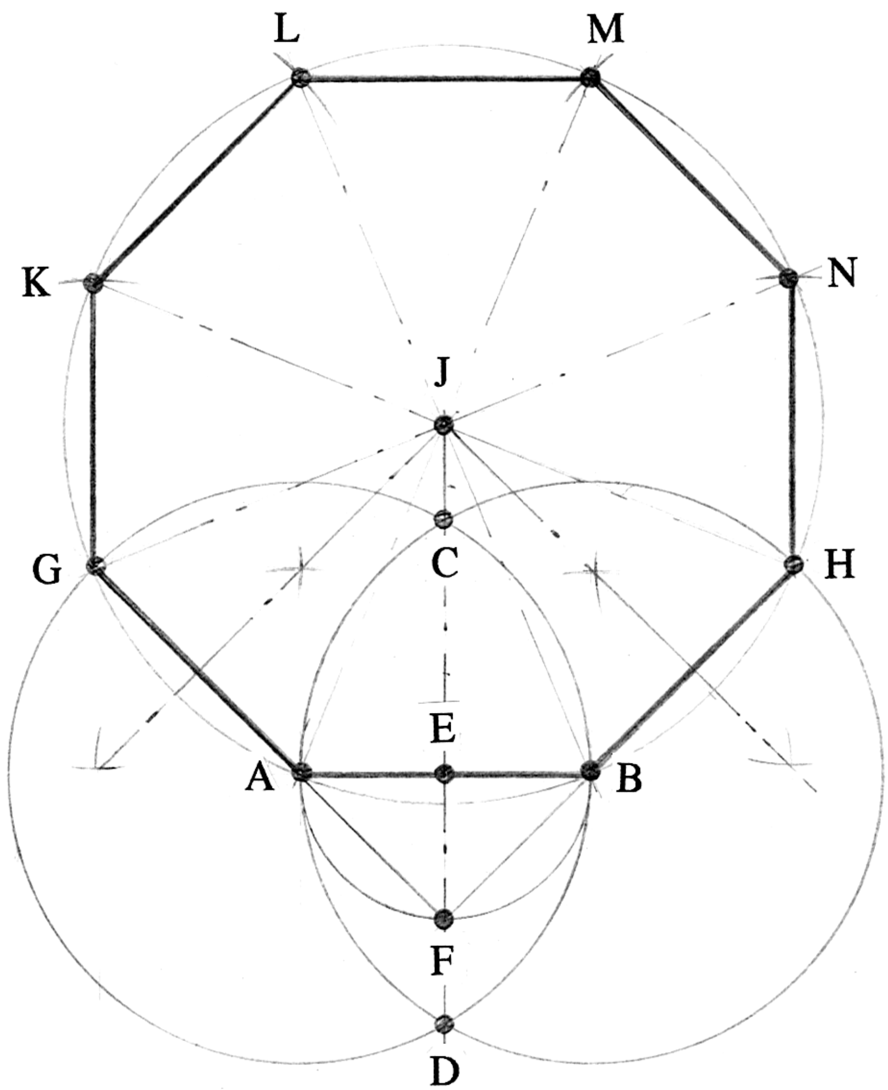
1. Given a circle with center at A.

2. Select any point on the circumference of the circle, point B.

3. Draw arc with center at B, radius AB to find points C and D.

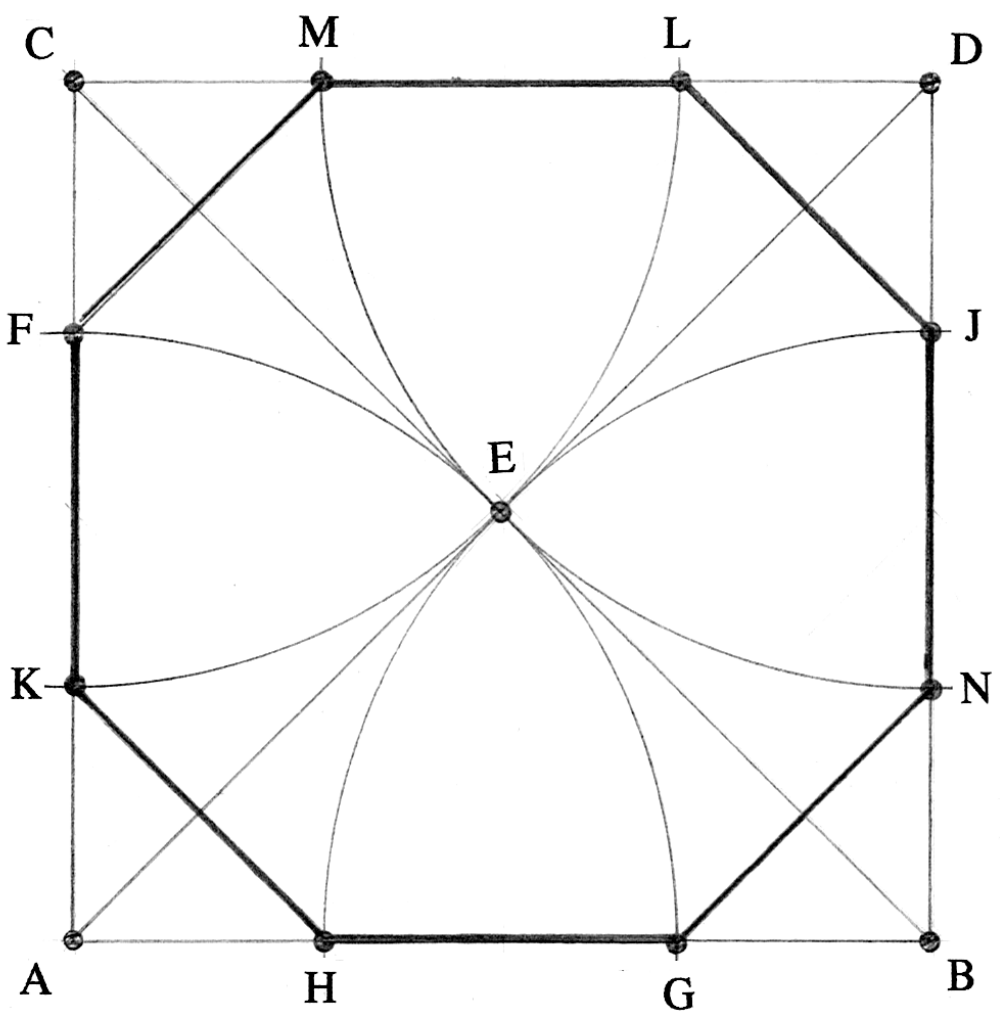
4. Draw arcs with centers at C and D, radius AB to find points E and F; and with centers at E and F to find G. BCDEFG is a hexagon.

3.3 Construct an octagon given a side.



1. Given line AB.
2. Draw circles with centers at A and B, radius AB to find points C and D. Draw line CD to find point E.
3. Draw arc with center at E, radus AE to find point F. Draw line AF and project to G; BF project to H.
4. Bisect AG [1.1] and project the bisector to find J on extension of CD.
5. Draw circle with center at J, radius AJ Lay off intervals of AB to find KLMN.
6. Alternately, project AJ to find M, BJ to find L, GJ to find N, HJ to find K. ABGHKLNM is an octogon

3.4 Construct an octagon within a square.

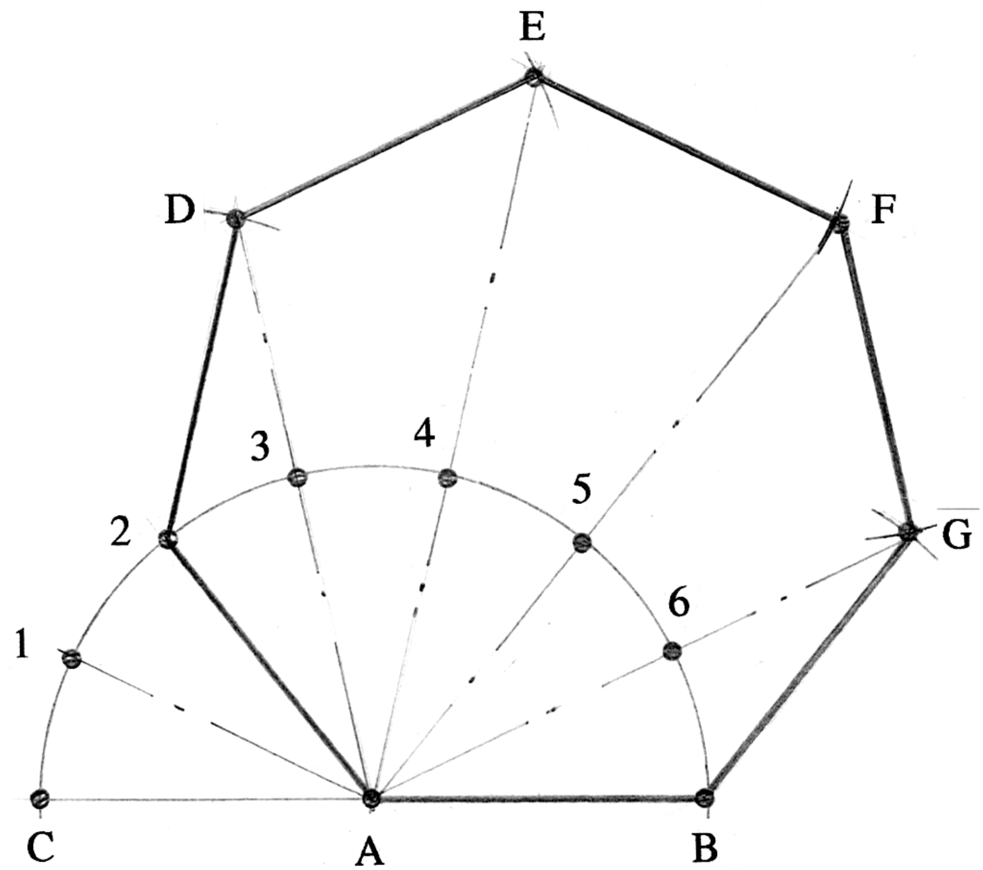


1. Given square ABCD.

2. Draw diagonals AD and BC to find point E.

3. Draw arcs with centers at A, B, C and D, radius AE to find F, G, H, J, K, L, M and N. FGHJKLMN is an octagon.

3.5 Construct an approximate heptagon given a side.



1. Given line AB.

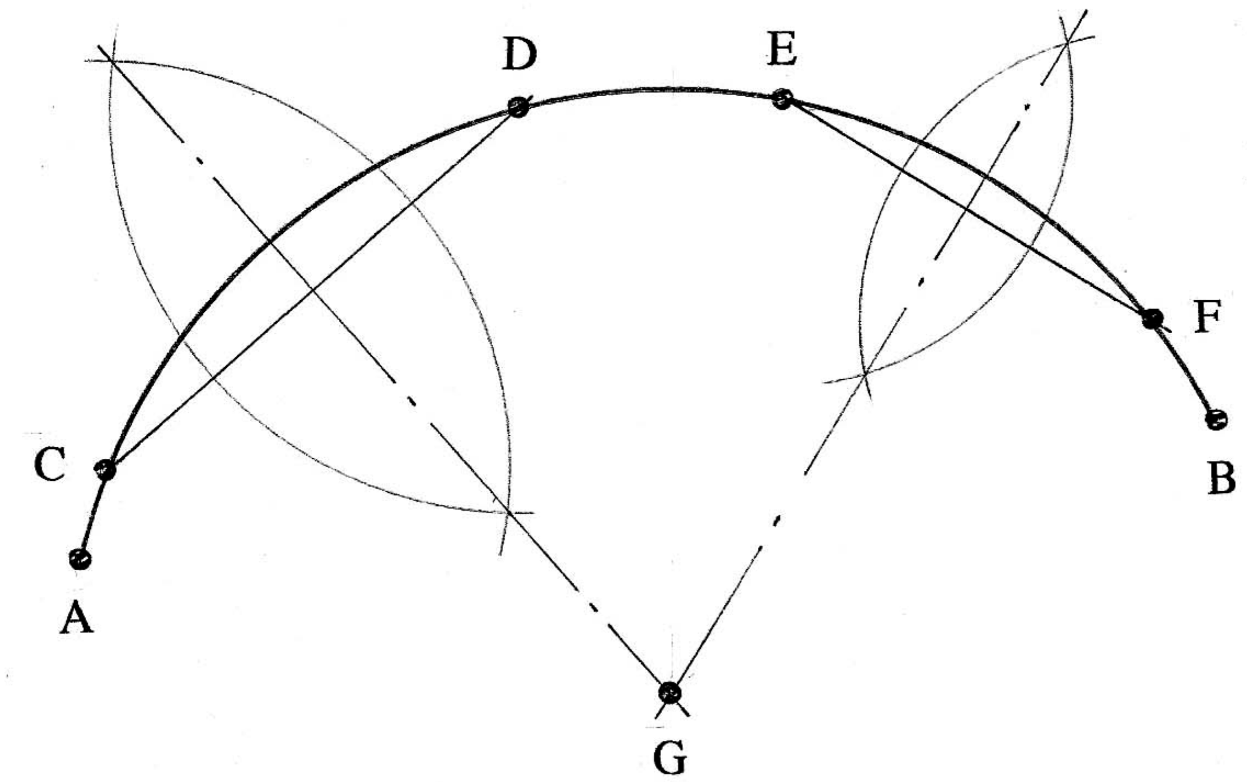
2. Draw a circle with center at A, radius AB
   Extend AB to point C.

3. Divide semi-circle CB into seven equal parts
   by trial and error to find points 1 through 6.

4. Draw line A2 and diagonals A3 through A6.

5. Draw arc with center at 2, radius AB to find
   point D. Repeat with center at D to find E, with
   center at E to find F, and with center at F to
   find G. AB2DEFG is a heptagon.

4.1 Find the center of a given arc.



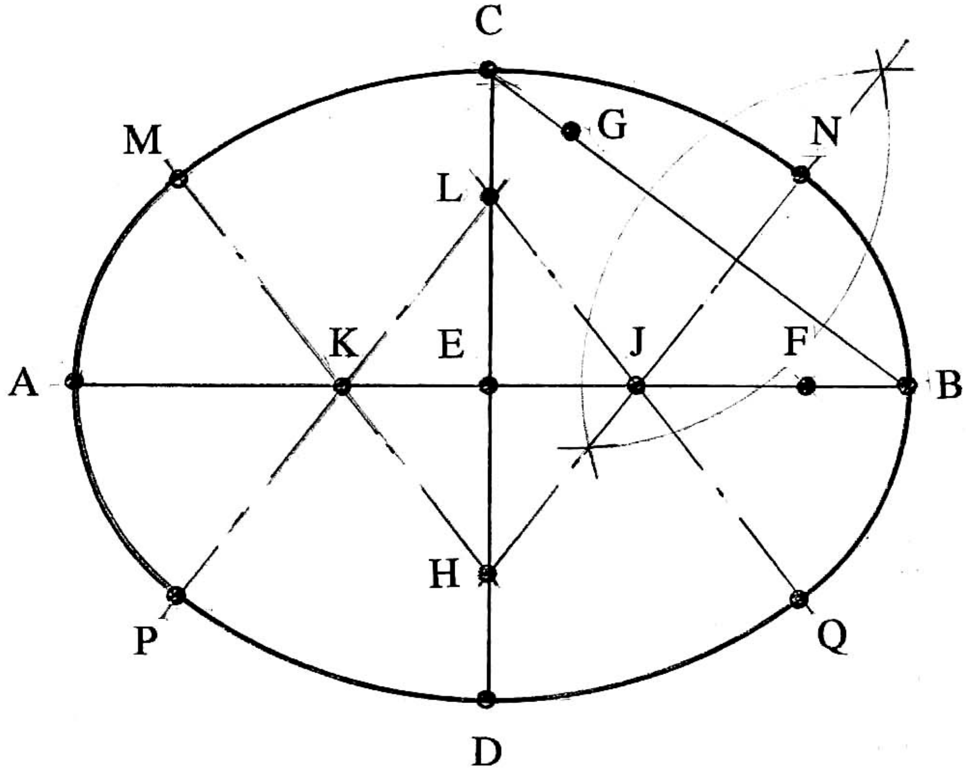
1. Given arc AB.

2. Draw any two chords CD and EF.

3. Bisect CD and EF. [1.1]

4. Extend the bisectors to find point G.
   G is the center of the circular arc.

4.2 Construct an approximate 4-centered ellipse given the major and minor axis.



1. Given major axis AB and minor axis CD with center at E.

2. Draw diagonal CB.

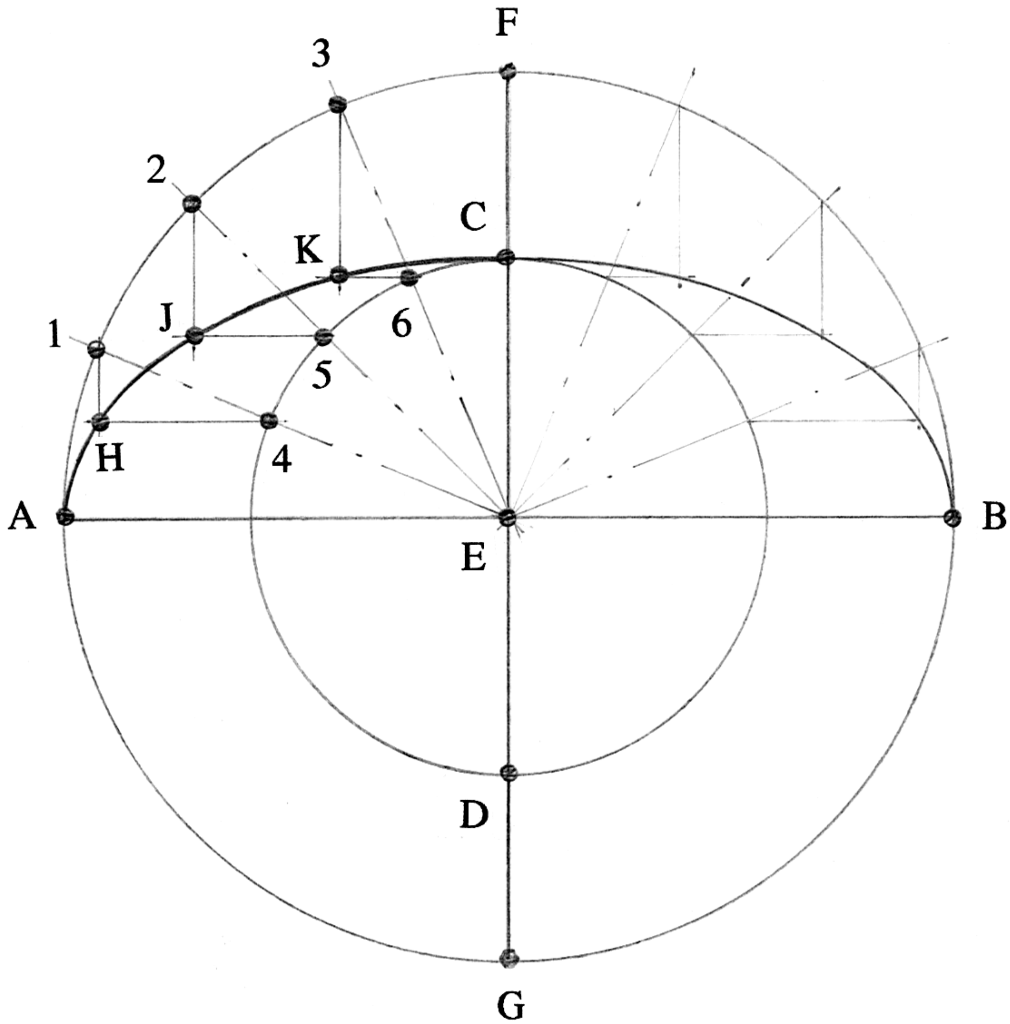
3. Lay off interval CE on EB to find point F. Translate FB to CB to find point G.

4. Bisect GB. [1.1] and extend the bisector to find points H and J. Mark off EK = EJ and EL = EH.

5. Draw an arc with center at H, radius HC to find M and N. and with center at L, radius LD = HC tofind P and Q

6. Draw an arcc with center at J, radius JB to be tangent at N and Q. Repeat with center at K, radius AK = JB to be tangent at M and P. ABCDMNPQ is an approx-imate ellipse.

4.3 Construct an approximate ellipse by the perpendicular method

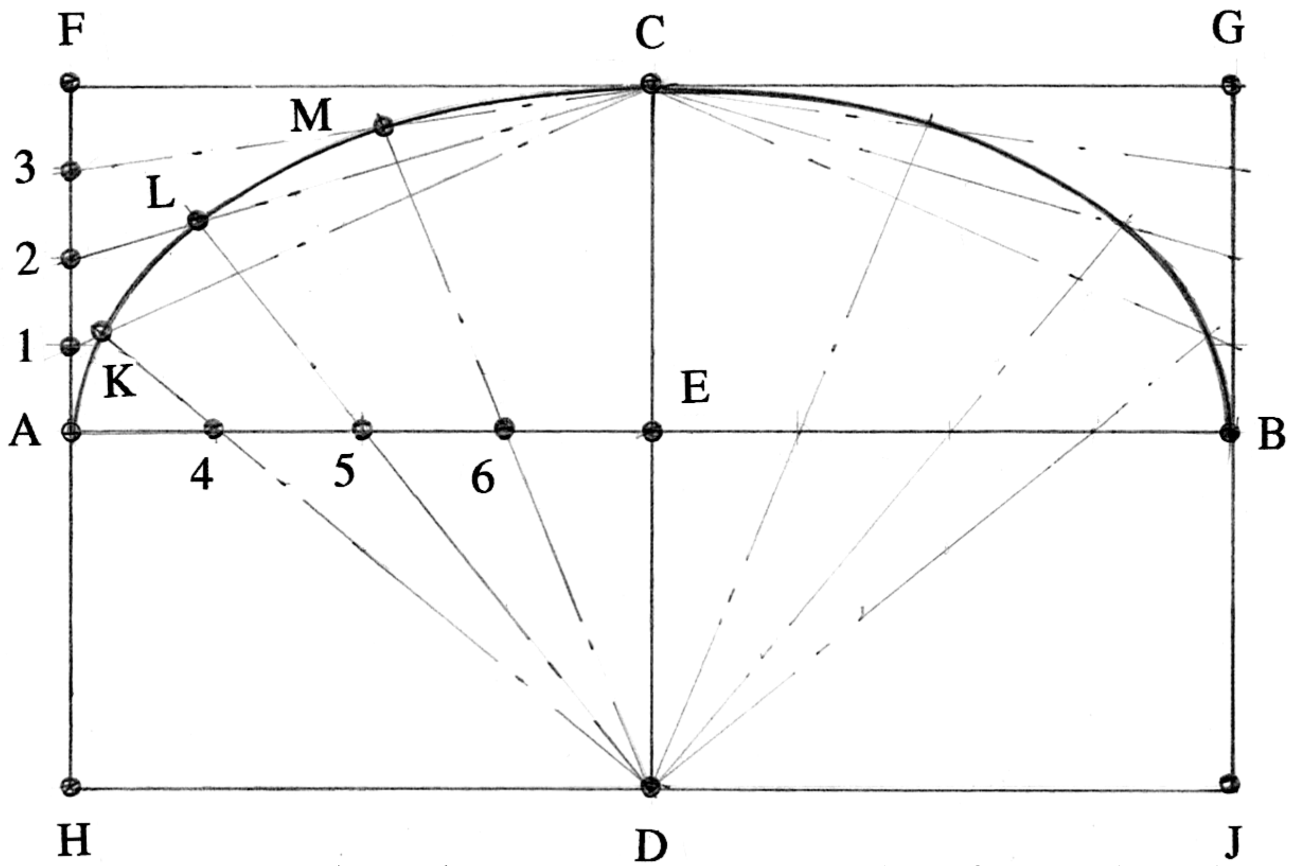


given the major and minor axis.

1. Given major axis AB and minor axis CD with center at E.
2. Draw two circles with center at E and radii EC and EA. Extend CD to find F and G.
3. Divide arc AF into a convenient number of equal parts, in this example four, to find points 1 - 6.
4. Project lines from 1, 2 and 3 perpendicular to AB. [1.3]

   Project lines from 4, 5 & 6 parallel to AB [1.5] to find H, J & K.
5. Draw a smooth curve through H, J & K, tangent to circles at A & C.
6. Repeat in remaining quadrants to complete the ellips

4.4 Construce an approximate ellipse by the diagonal method given the major and minor axis.



1. Given major axis AB and minor axis CD, intersecting at E.

2. Construct rectangle FGHJ. [similar to 2.1]

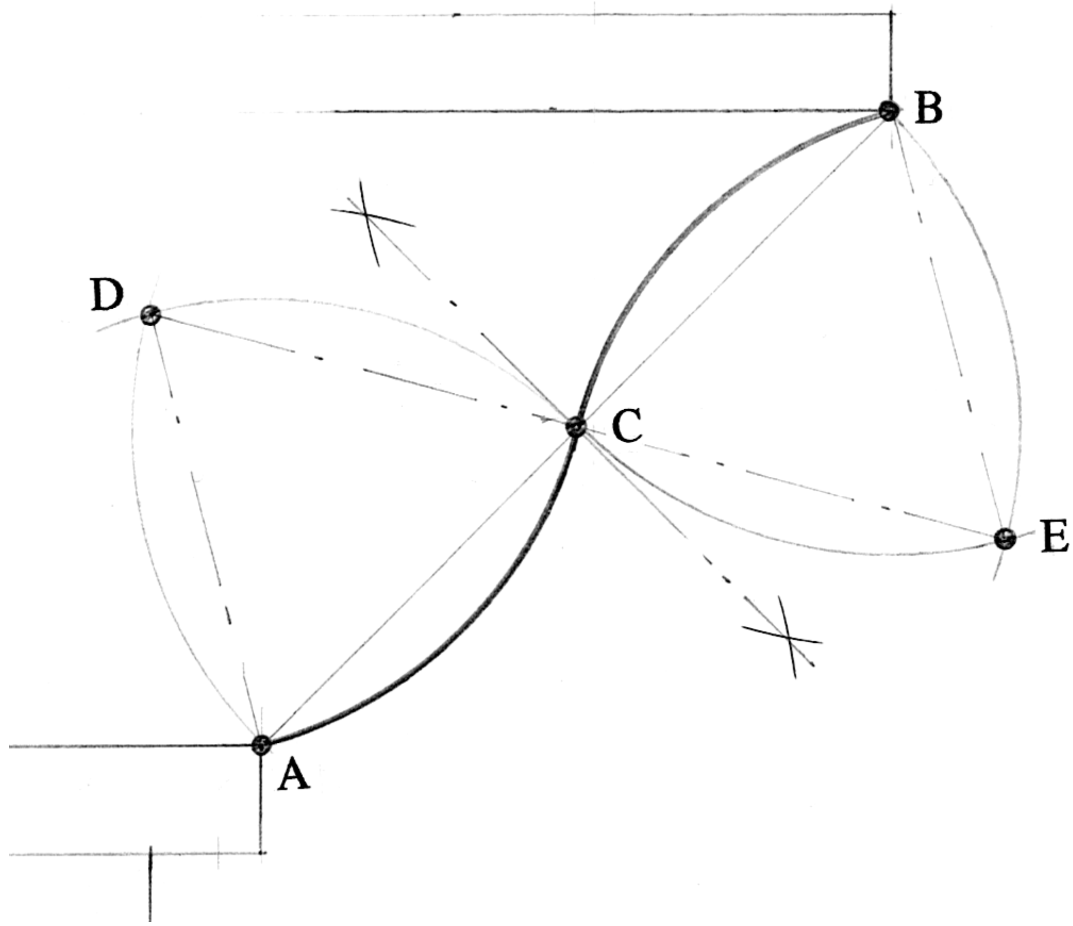
3. Divide AE & AF into the same number of equal parts, in this example 4, to find points 1 - 6.

4. Project lines from C to points 1, 2, and 3.

5. Project lines from D through points 4, 5 and 6 to intersect the projectors from C at points K, L and M.

6. Draw a smooth curve through H, J and K, and tangent at A and C. Repeate in the other quadrents to complete the ellipse.

5.1 Construct a cyma recta profile.



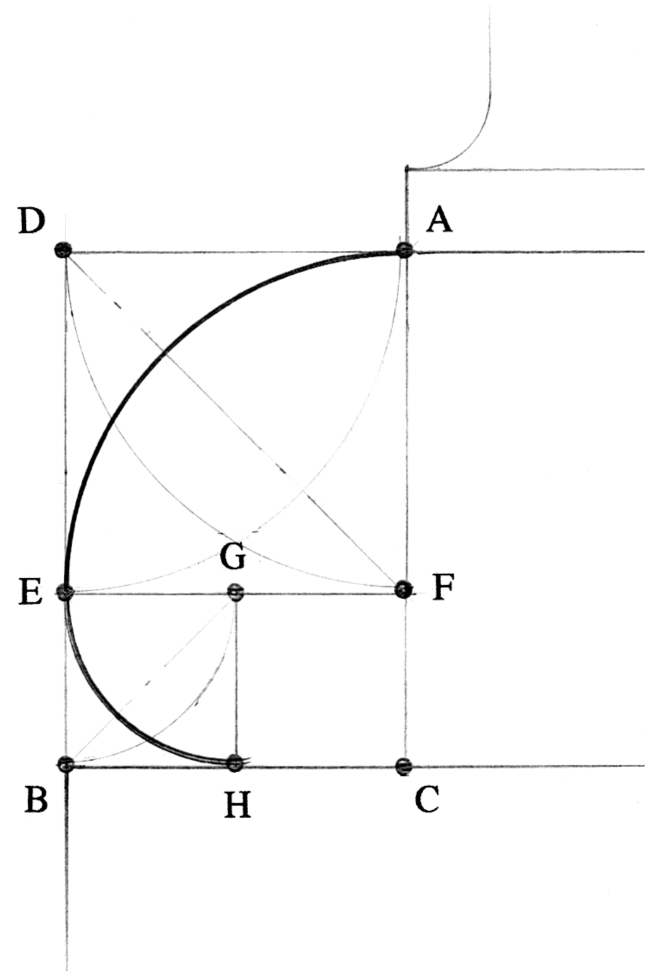
1. Given points A and B.

2. Draw line AB and bisect [1.1] to find point C.

3. Construct equilateral triangles ACD and CBE. [1.6]

4. Draw arcs with centers at D and E, radius AC = CB, tangent at C. Curve ACB is a cyma recta profile.

5.2 Construct a torus profile using two quarter-circles.



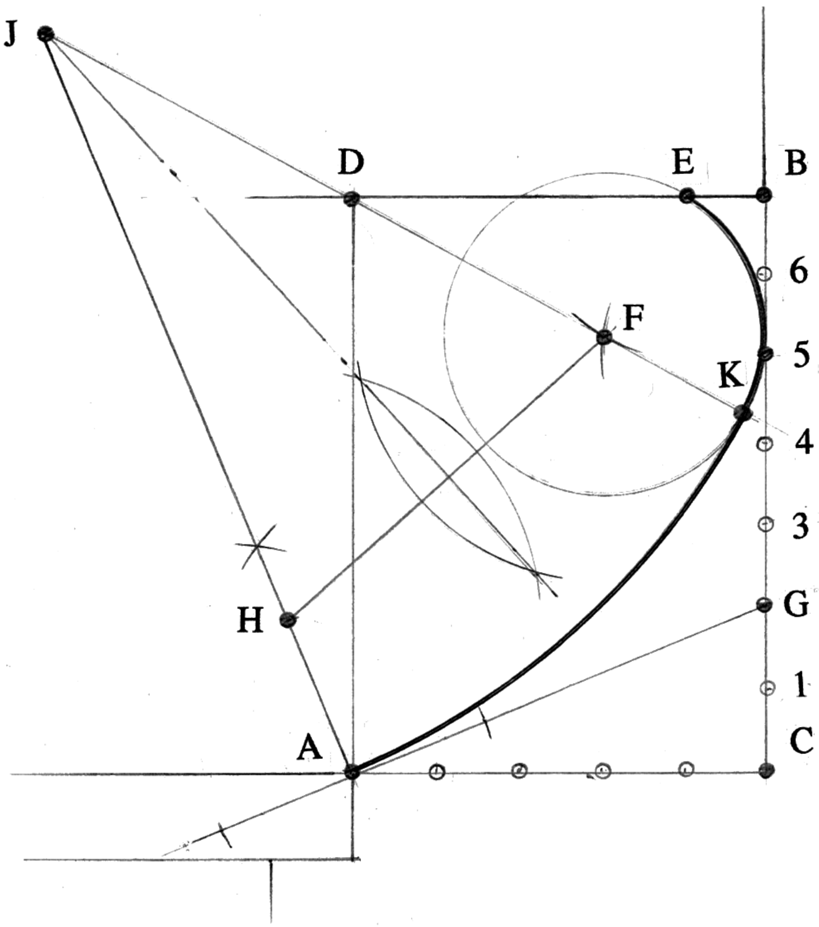
1. Given points A and B.

2. Construct rectangle ABCE. [2.1 similar]

3. Construct squares ADEF and BEGH. [2.1]

4. Draw arc with center at F, radius AF. Draw arc with center at G, radius EG, tangent at E. Curve AEH is the profile.

5.3 Construct a quirked ovolo profile - circular arc method.



1. Given points A and B.

2. Construct rectangle ABCD. In this example AC = 5 and BC = 7 units.

3. Lay off 1 unit along BD to find point E.

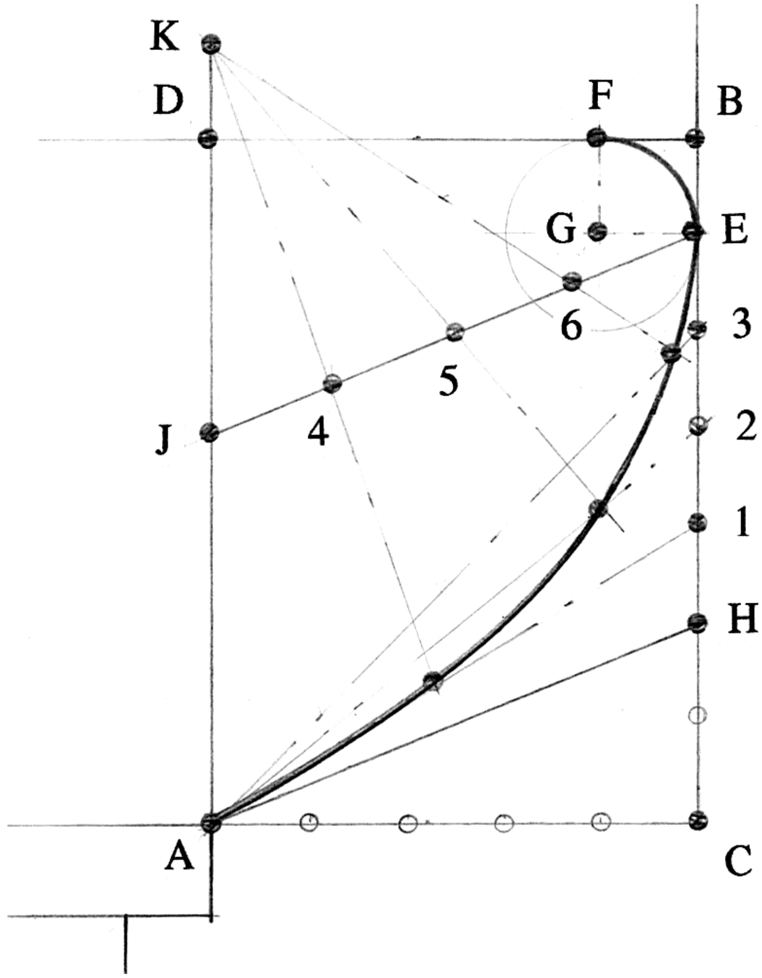
4. With centers at E and 5 strike arcs of radius = 2 units to find F. Draw a circle, center at F, radius = 2 units.

5. Draw line AG. Construct a perpendicular on point A. [1.2] Lay off 2 units along the perpendicular to find point H.

6. Draw line HF. Bisect HF [1.1], project the bisector to intersect the perpendicular on A to find J.

7. Draw an afc with center at J, radius JA, tangent at K . Curve AEK is the profile.

5.4 Construct a quirked ovolo - hyperbolic method.



1. Given points A and B.

2. Construct rectangle ABCD. [2.1 similar] In this example AC = 5, CB = 7 equal units.

3. Construct square BEFG, BE = 1 unit.

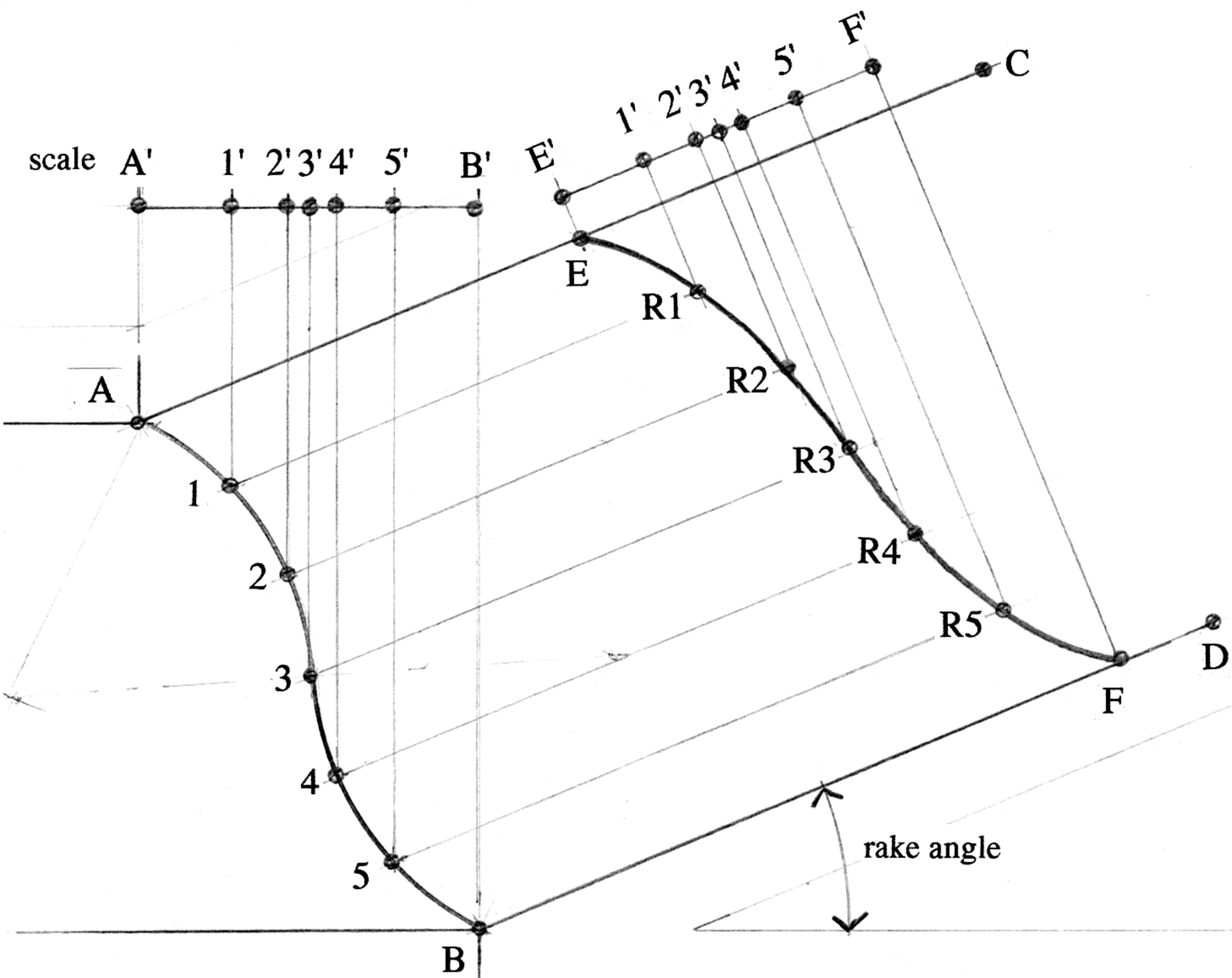
4. Draw lines AH, A1, A2 and A3.

5. Lay off AJ = EH. Draw line EJ and divide into 4 equal parts to find points 4, 5 and 6.

6. Extend AD to find point K, KJ = JA. Project lines from K through 4, 5 and 6 to intersect lines A1, A2 and A3 respectively.

7. Draw a smooth curve through the intersections, tangent at A and E. AEF is the ovolo profile.

5.5 Construct the raking profile of a cyma recta moulding.



1. Given horizontal cyma recta moulding AB and raking lines AC & BF.

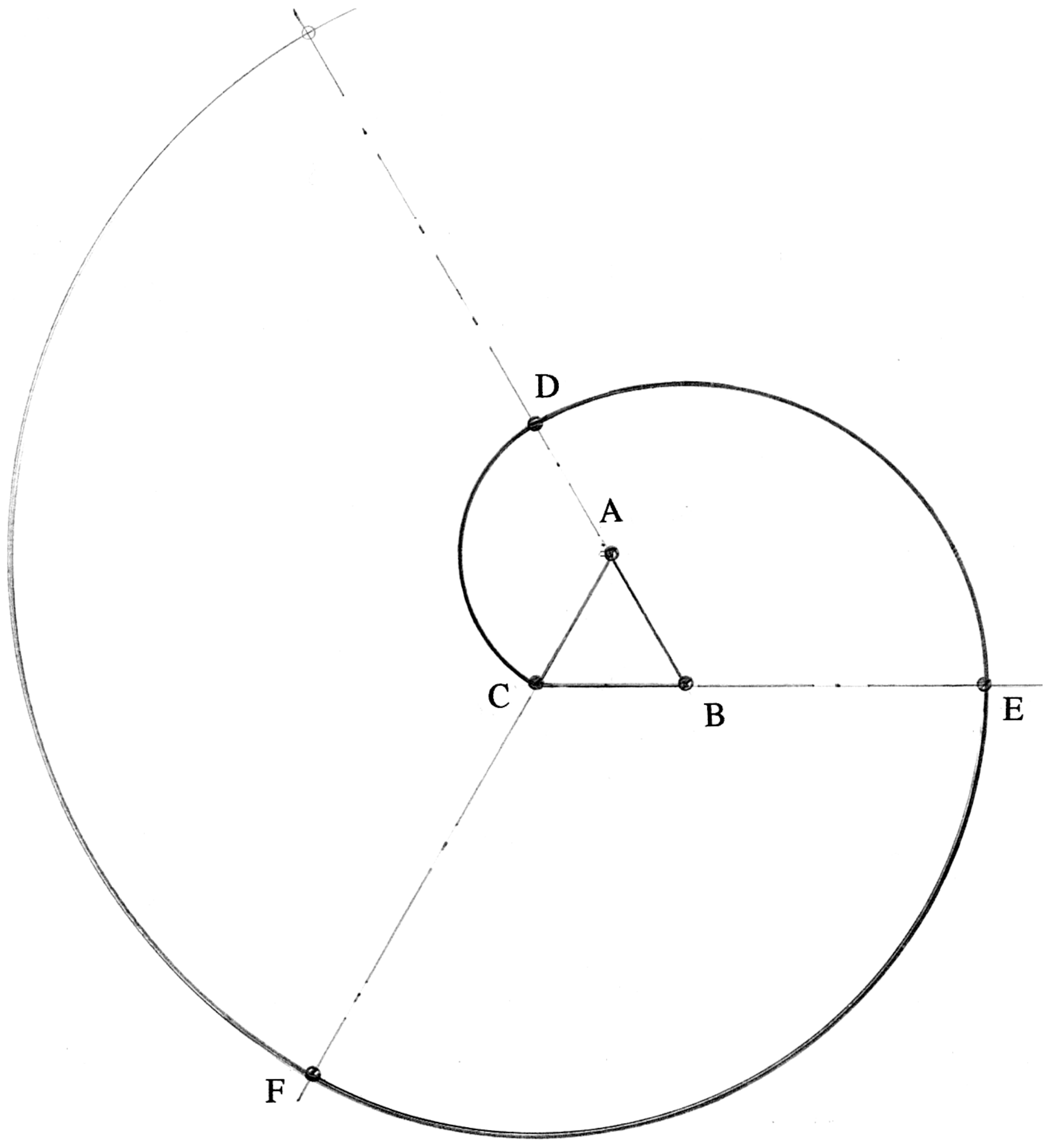
2. Divide profile AB into a number of equal parts, in this example six, to find points 1 thru 5.

3. Draw perpendiculars from A, 1-5 and B [1.3] to establish scale A'B'.

4. Translate scale A'B' to convenient point E. Lay off E'F' = A'B' perpendicular to the rake angle AC & BD.

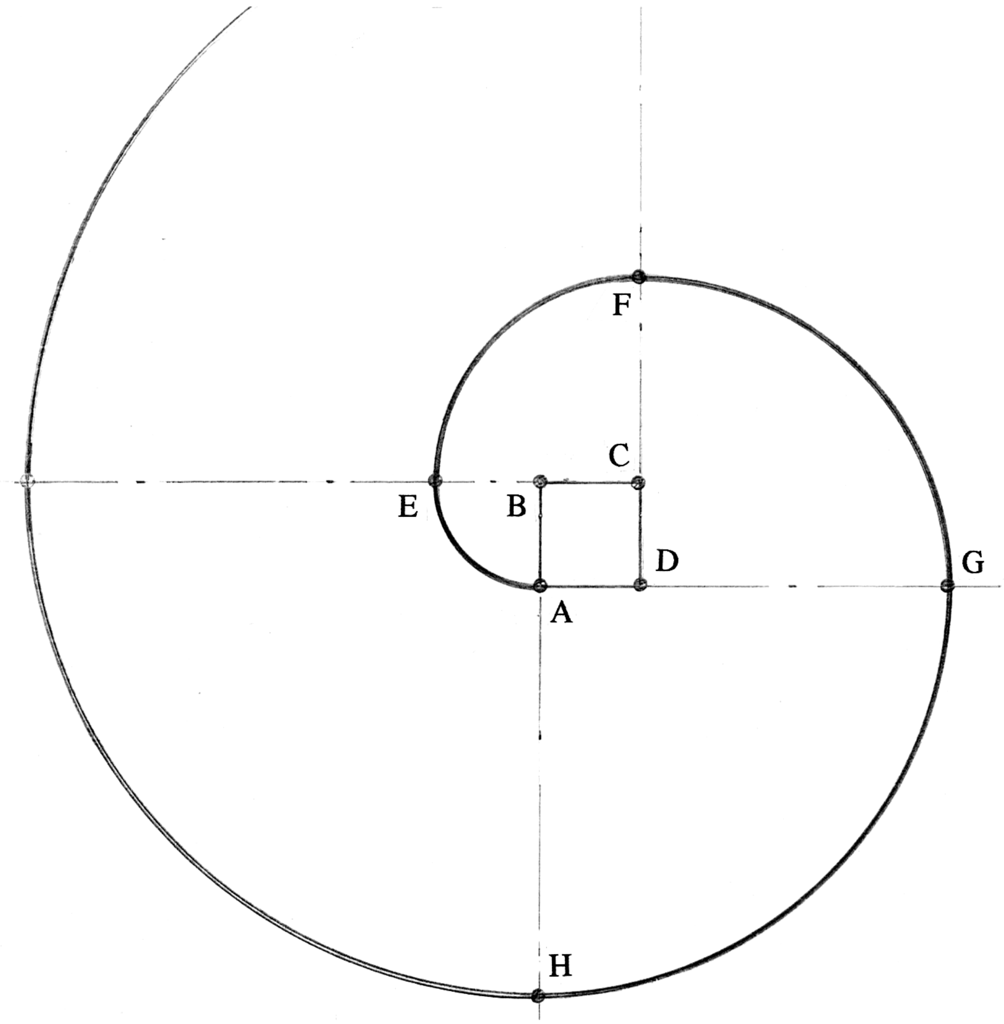
5. Project perpendiculars through points 1' - F' to intersect parallels 1 - 5 to find points R1 - R5 & F. A smooth curve through points E - F are the raking cyma profile.

6.1 Construct a spiral on an equilateral triangle



1. Given equilateral triangle ABC.
2. Extend AB, BC, and AC.
3. Draw an arc with center at A, radius AC to find point D.
4. Draw an arc with center at B, radius BD to find point E.
5. Repete with center at C to find F. Continue for as many segments as desired. CDEF is the spiral.

6.2 Construct a spiral on a square.



1. Given square ABCD.

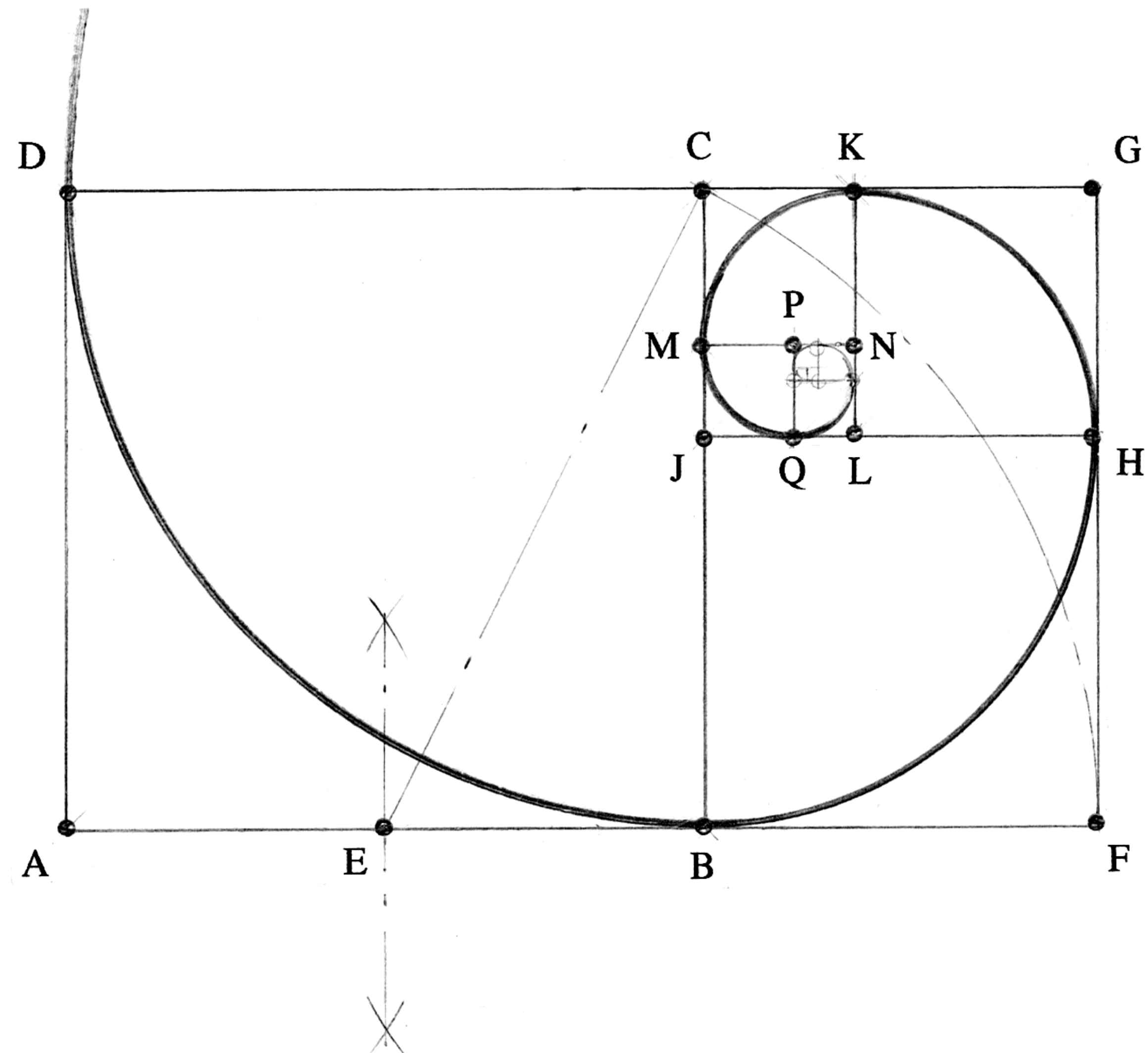
2. Extend each side.

3. Draw an arc with center at B, radius AB to fiind point E.

4. Draw arcs with center at C, radius EC to find F; with center at D, radius DF to find G; with center at A, radius AG to find H.

5. Continue as desired. AEFGH is the spiral.

6.3 Construct a Golden Section spiral.



1. Given square ABCD.

2. Bisect AB to find E. [1.1] Lay off radius ED on the extension of AB to find F. [2.3] AB:AF = 1:1.618.

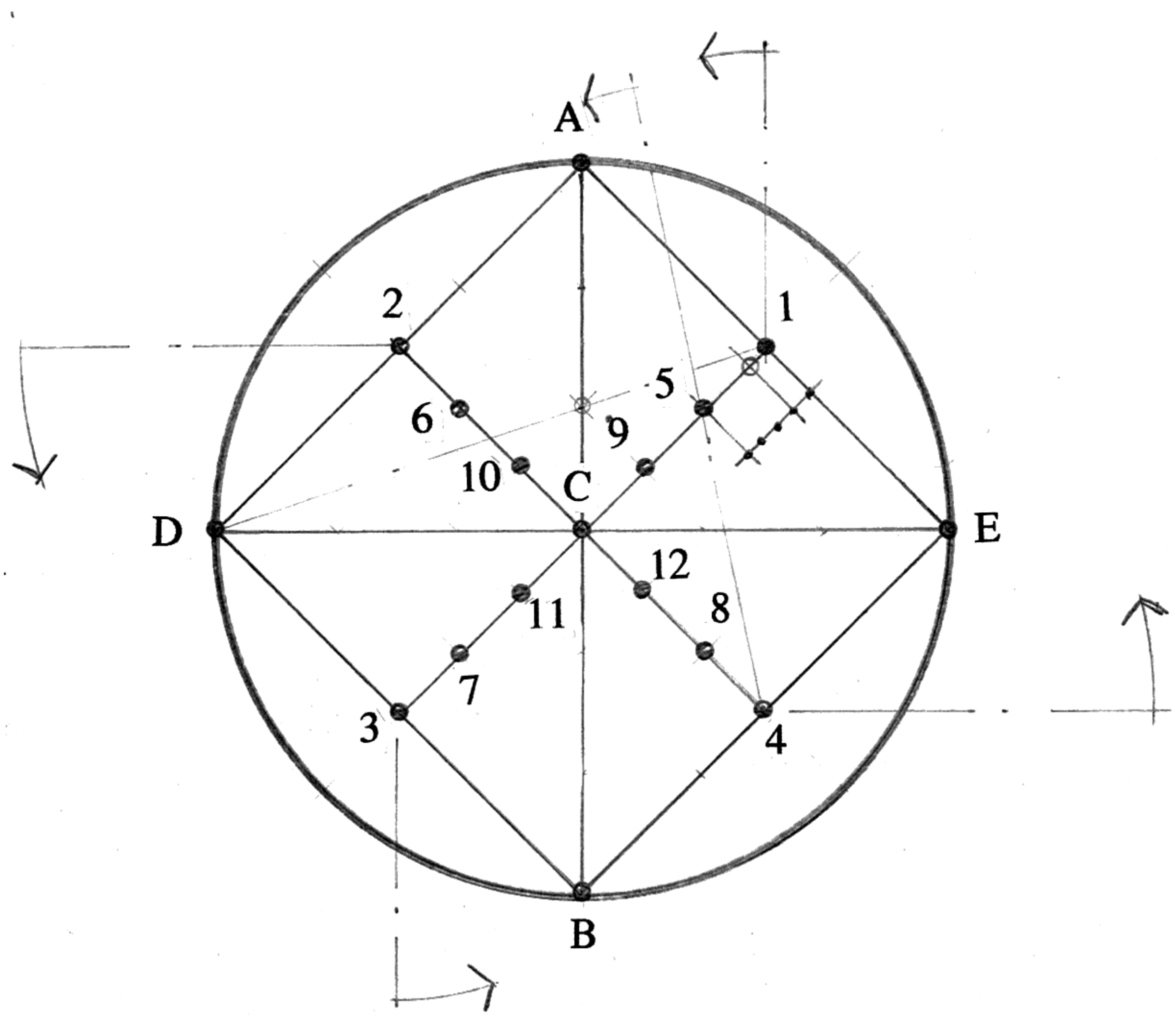
3. Construct a perpendicular on F [1.2] ACFG is a Golden Section rectangle.

4. Draw squares BFJH, GHKL, CKMN and JMPQ. [2.1]

5. With center at C draw arc DB; with center at J draw arc BH; with center at L draw arc HK.

6. Continue as desired. DBHKMQ is a Golden Section spiral.

6.4 Construct the eye of the Ionic volute - Vignola's method.



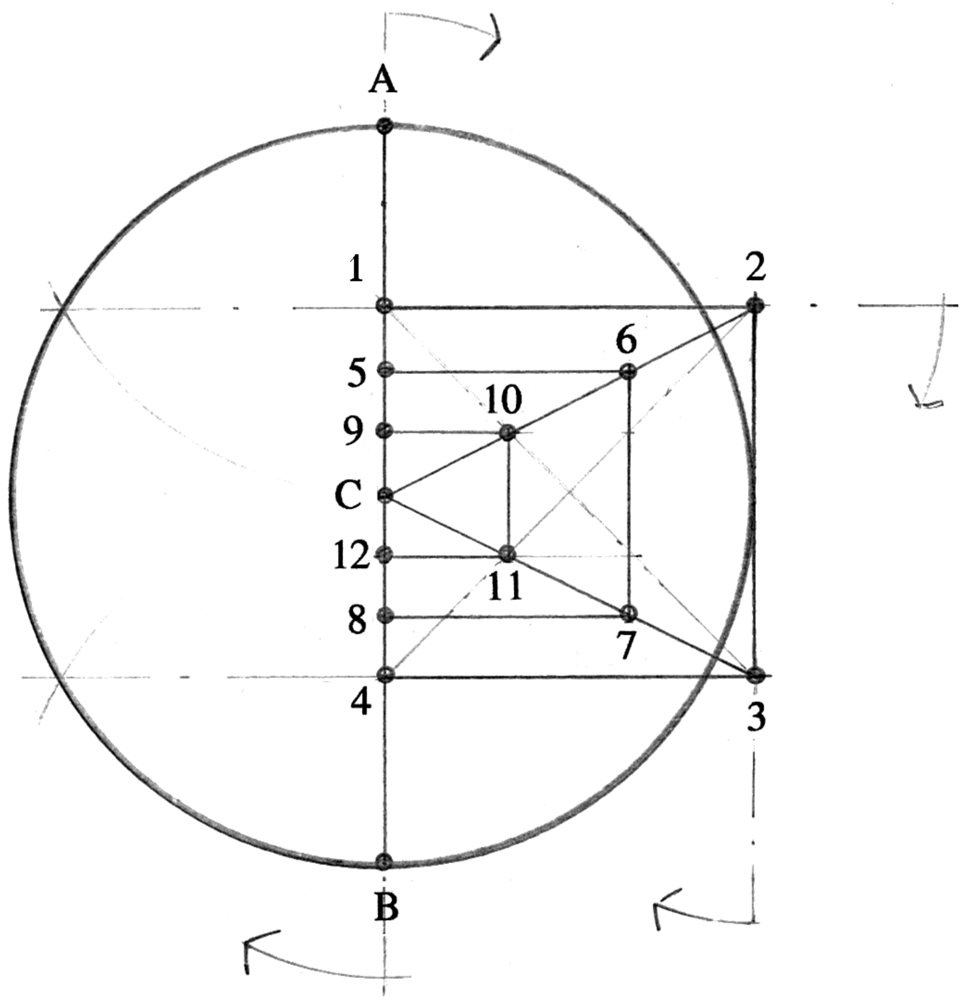
1. Given circle with diameter AB, center at C.

2. Draw diameter DE, perpendicular to AB. Draw square AEBD. [1.2]

3. Bisect the sides of the square [1.1] to find points 1 - 4. Divide 1C through 4C into three to find points 5 - 12. Points 1 - 12 are the centers of the volute segments.

4. To find the centers of the fillet divide interval 1,5 into four parts. The first part gives the center for the fillet. Repeat at each of the segments.

6.5 Construct the eye of the Ionic volute - Goldman's method



1. Given circle with diameter AB, center at C.

2. Bisect AC and CB [1.1] to find points 1 and 4.

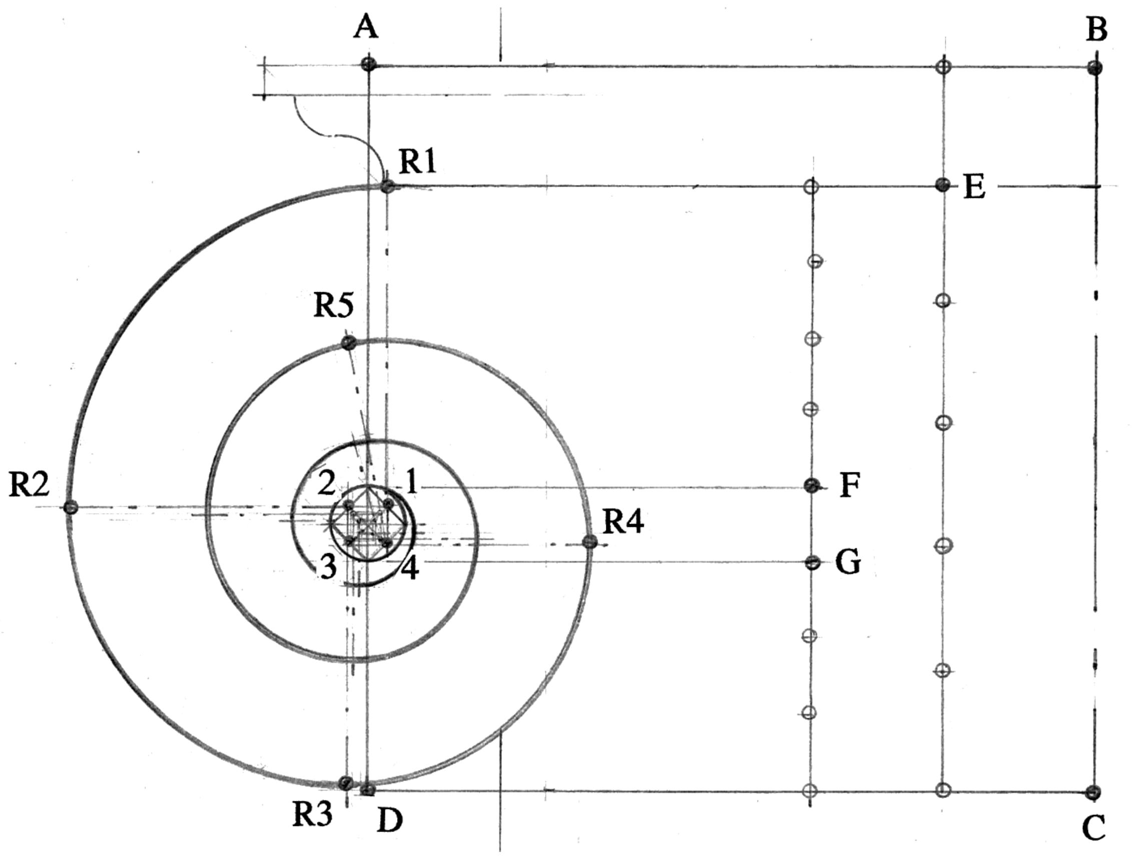
3. Construct a square [2.1] with side 1

   1,4 = AC to find points 2 and 3.

   Line 2,3 is tangent to circle AB

4. Divide 1C into three equal parts and draw squares 5,6,7,8 and 9,10,11,12. Points 1 - 12 are the centers of the volute segments.

6.6 Construct the Ionic volute - Vignola's method



1. Given square ABCD, AB = 1/2 of lower column diameter.

2. Divide AD into six parts. Mark off the top sixth, point E to find point R1. Divide length R1,D into eight parts to find points F and G.

3. To place the eye draw a circle with diameter FG, center on AD.

4. Place the 12 volute centers as per 6.4.

5. Draw an arc with center at 1 and radius 1,R1. Stop at the projection of line 1,2 to find R2. Draw an arc with center at 2, radius 2,R2 to find R3.

6. Continue with center at 3, radius 3,R3, find R4; with center at 4, radius 4,R4, find R5. R5 is the projection of centers 4 and 5.

7. continue through the 12 centers for 3 complete rotations of the spiral. The spiral line should meet the circumference of the eye at line 1,R1. This completes the volute.

End of Constructions